Distribution systems fault analysis considering fault resistance estimation

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Abstract

Fault resistance is a critical component of electric power systems operation due to its stochastic nature. If not considered, this parameter may interfere in fault analysis studies. This paper presents an iterative fault analysis algorithm for unbalanced three-phase distribution systems that considers a fault resistance estimate. The proposed algorithm is composed by two sub-routines, namely the fault resistance and the bus impedance. The fault resistance sub-routine, based on local fault records, estimates the fault resistance. The bus impedance sub-routine, based on the previously estimated fault resistance, estimates the system voltages and currents. Numeric simulations on the IEEE 37-bus distribution system demonstrate the algorithm’s robustness and potential for offline applications, providing additional fault information to Distribution Operation Centers and enhancing the system restoration process.

1. Introduction

Electric power systems are daily exposed to service interruption mainly due to faults and human accidental interference. A power system fault is defined as any failure which interferes with the normal current flow [1]. The fault phenomenon affects system’s reliability, security, and energy quality, and can be considered stochastic. Different events such as lightning, insulation breakdown and trees falling across lines are common overhead power system fault causes.

Power system faults may be classified as temporary or permanent. Temporary faults in overhead lines are usually caused by lightning. In this case, system service can be automatically restored after approximately 20 fundamental frequency cycles, with the circuit breakers opened, to allow deionization. However, permanent faults are associated with different events, like trees falling across lines. On these situations, system restoration is maintenance crew dependent. Facility maintenance crew must search and repair the system using a fault location estimate. For non-negligible fault resistances ($R_f$), which are commonly associated with permanent faults, standard fault location algorithms may present poor performance [2,3].

Fault analysis methods are an important tool used by protection engineers to estimate power system currents and voltages during disturbances. It provides information for protection system setting, coordination and efficiency analysis studies. Today, three approaches are used in the industry for such analysis: classical symmetrical components, phase variable approach and complete time-domain simulations [4]. Classical fault analysis of unbalanced power systems is based on symmetrical components approach [5,6]. However, in untransposed feeders with single-phase or double-phase laterals, the symmetrical component methods do not consider accurately these specific characteristics [7]. Hence, symmetrical components based techniques may not provide accurate results for power distribution systems, which are normally characterized by those asymmetries.

With industrial computer facilities improvement, the fault analysis phase variable approach has been proposed to substitute the symmetrical components methods on distribution systems [8]. In the phase variable approach, system voltages and currents are related through impedance and admittance matrices based on phase frame representation, considering the typical distribution systems asymmetries.

However, fault analysis is still fault resistance dependant [9]. Due to fault resistance stochastic nature, typical fault analysis studies consider the fault paths as an ideal short-circuit. To overcome this limitation, recent studies suggest the usage of fault resistance estimation algorithms [10–12]. These works provide a fault resistance estimate using symmetrical components or modal analysis techniques, restricting the application on balanced systems.
with equally transposed lines. The usage of artificial intelligence has also been recently proposed in order to overcome the fault resistance effects in classical power system protection [13] and fault location [14] applications.

Considering the above mentioned limitations of state-of-the-art fault analysis methods, this paper proposes an iterative fault analysis algorithm that considers typical distribution systems characteristics, and a fault resistance estimate. The proposed fault analysis algorithm is composed by two sub-routines, namely the fault resistance and the bus impedance sub-routines. The fault resistance sub-routine is based on an iterative formulation executed to estimate the fault resistance through one-terminal fault records. The bus impedance sub-routine considers the estimated fault resistance values, and uses a bus impedance matrix based formulation to estimate the fault system voltages and currents.

In order to validate the proposed fault analysis algorithm, the formulation was implemented in MATLAB [15] and had its performance evaluated using a modified IEEE 37 Node Test Feeder [16], simulated under BPA’s ATP/EMTP software [17].

The remaining of this paper is organized as follows. The second section presents the fault resistance sub-routine. Section 3 describes the bus impedance sub-routine. A case study is presented in Section 4. Finally, Sections 5 and 6 discuss the results and conclusions obtained from this work.

2. Fault resistance sub-routine

Fault resistance represents the fault impedance path between phase or ground faults [18]. The proposed fault resistance sub-routine uses as input data, the sending-end voltages and currents. In the following subsections the fault resistance sub-routine is presented.

2.1. Mathematical development

Referring to the power system of Fig. 1, the sending-end voltages are given by (1), which describes the steady-state fault conditions:

\[
\begin{align*}
V_{Sfm} &= x \cdot \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{a0} & z_{bb} & z_{bc} \\ z_{a0} & z_{b0} & z_{cc} \end{bmatrix} \begin{bmatrix} I_{Sfa} \\ I_{Sfb} \\ I_{Sfc} \end{bmatrix} + \begin{bmatrix} V_{fa} \\ V_{fb} \\ V_{fc} \end{bmatrix} \\
V_{Sfm} &= \begin{bmatrix} V_{Sfp} \\ V_{Sfb} \\ V_{Sfc} \end{bmatrix}
\end{align*}
\]

where \(V_{Sfp}\) is the phase \(m\) sending-end voltage (V); \(x\) the distance between the sending-end and the fault location (m); \(z_{mn}\) the phase \(m\) line self impedance (\(\Omega/m\)); \(z_{mn}\) the mutual impedance between phases \(m\) and \(n\) (\(\Omega/m\)); \(I_{Sfm}\) the phase \(m\) sending-end current (A); \(V_{Sfm}\) the phase \(m\) fault location voltage (V); \(m\) is the phases a, b, or c.

For the single-line-to-ground fault (SLG) illustrated in Fig. 1, the faulted phase sending-end voltage from (1) can be expanded to (2):

\[
V_{Sfp} = V_{fp} + x \cdot (z_{pa} \cdot I_{Sfa} + z_{pb} \cdot I_{Sfb} + z_{pc} \cdot I_{Sfc})
\]

where

\[
V_{fp} = Z_F \cdot I_{fp}
\]

whereas \(Z_F\) is the fault impedance between line-to-ground, the subscript \(p\) represents the faulted phase, and \(I_{fp}\) is the phase \(p\) fault current.

Considering the fault impedance strictly resistive and constant, (2) may be expanded into its real and imaginary parts:

\[
\begin{align*}
\begin{bmatrix} V_{Sfp} \\ V_{Sfb} \\ V_{Sfc} \end{bmatrix} &= \begin{bmatrix} M_{1p} & I_{Fp} \\ M_{2p} & I_{Fp} \\ M_{3p} & I_{Fp} \end{bmatrix} \cdot \begin{bmatrix} x \\ R_F \end{bmatrix} \\
M_{1p} &= \sum_{k=(a,b,c)} (|z_{pk}|^2 - I_{Sfkm} \cdot I_{Sfkm}) \\
M_{2p} &= \sum_{k=(a,b,c)} (|z_{pk}|^2 + I_{Sfkm} \cdot I_{Sfkm}) \\
\end{align*}
\]

where the subscripts \(r\) and \(i\) represent the real and imaginary components, \(R_F\) is the fault resistance, and:

\[
\begin{align*}
VF_k &= \sum_{k=(a,b,c)} (z_{pk} \cdot I_{Sfkm} - z_{pk} \cdot I_{Sfkm}) \\
VF_k &= \sum_{k=(a,b,c)} (z_{pk} \cdot I_{Sfkm} + z_{pk} \cdot I_{Sfkm})
\end{align*}
\]

where \(z_{pk}\) is the mutual impedance between the faulted phase \(p\) and non-faulted phase \(k\) (\(\Omega/m\)).

Evaluating (4), the fault distance and resistance may be calculated as function of the sending-end voltages and currents, as well as the line parameters, as given by (7):

\[
\begin{align*}
\begin{bmatrix} x \\ R_F \end{bmatrix} &= \begin{bmatrix} M_{1p} & I_{Fp} \\ M_{2p} & I_{Fp} \end{bmatrix}^{-1} \begin{bmatrix} I_{Fp} \\ I_{Fp} \end{bmatrix} \\
V_{Sfp} &= M_{1p} \cdot I_{Fp} - M_{2p} \cdot I_{Fp} \\
V_{Sfb} &= M_{2p} \cdot I_{Fp} - M_{1p} \cdot I_{Fp} \\
V_{Sfc} &= M_{3p} \cdot I_{Fp} - M_{1p} \cdot I_{Fp}
\end{align*}
\]

From (7), fault resistance and distance independent mathematical expressions for single line-to-ground faults are obtained, given by (8) and (9), respectively:

\[
\begin{align*}
x &= \frac{V_{Sfp} \cdot I_{Fp} - V_{Sfb} \cdot I_{Fp}}{M_{1p} \cdot I_{Fp} - M_{2p} \cdot I_{Fp}} \\
R_F &= \frac{M_{1p} \cdot V_{Sfp} - M_{2p} \cdot V_{Sfc}}{M_{1p} \cdot I_{Fp} - M_{2p} \cdot I_{Fp}}
\end{align*}
\]

As a result of the procedure described above, the fault distance and resistance may be estimated. To obtain such estimates, the system parameters, sending-end voltages and currents should be known. The fault current is the only unknown variable on such expressions and is calculated by an iterative procedure, as presented in the following.

2.2. Fault current estimation

Referring to Fig. 1, the fault current \(I_{fp}\) may be estimated through the difference between the load current and the sending-end current, as given by (10):

\[
|I_f| = |I_{Sp}| - |I_l|
\]

where \(I_{Sp}\) is the sending-end three-phase current vector; \(I_l\) is the three-phase load current vector.

Considering the fault period load current different from the pre-fault load current, due to system dynamics, an iterative formulation is developed to estimate the first [19], as follows:

(1) Fault period load current is initially considered equal to the pre-fault load current.
(II) Fault current is calculated using (10).
(III) Fault location and resistance are determined by (8) and (9), respectively.
(IV) Fault location voltages are estimated through (11):
\[
\begin{bmatrix}
V_{Fp} \\
V_{Fq} \\
V_{Fr}
\end{bmatrix} =
\begin{bmatrix}
V_{Gp} \\
V_{Gq} \\
V_{Gr}
\end{bmatrix} - x \cdot
\begin{bmatrix}
Z_{ap} & Z_{ab} & Z_{ac} \\
Z_{bp} & Z_{bb} & Z_{bc} \\
Z_{cp} & Z_{cb} & Z_{cc}
\end{bmatrix} \cdot
\begin{bmatrix}
I_{Gp} \\
I_{Gq} \\
I_{Gr}
\end{bmatrix}
\]

(V) An equivalent admittance matrix between the load and the line impedance between the fault location and the receiving-end is calculated by (12):

\[Y_L = (I - x \cdot Z + |Z|)^{-1}\]

where \([Z]\) is the line impedance matrix per unit length, \([Z_L]\) is the load impedance matrix, and \(I\) is the line length.

(VI) The three-phase load current vector is updated using the equivalent admittance matrix obtained by (12) and the fault location voltages:

\[I_L = Y_L \cdot V_F\]

(13)

(VI) The algorithm verifies if the fault resistance and distance have converged, using (14) and (15):

\[|R_F(n) - R_F(n-1)| < \delta_1\]

(14)

\[|x(n) - x(n-1)| < \delta_2\]

(15)

where \(n\) is the iteration number and \(\delta_{1,2}\) are the error tolerances, which are previously defined according to the accuracy and computational time desired. In this work, \(\delta_1\) and \(\delta_2\) have been considered equal to \(1 \times 10^{-7}\) and \(1 \times 10^{-4}\), respectively.

(VI) If the fault resistance and distance have converged, stop the iterative procedure, otherwise return to step II.

Thus, the fault distance can also be used as a fault location estimate. However, as typical impedance-based fault location techniques, for higher fault resistance values the fault distance estimate may be inaccurate [19]. An additive error in the fault current estimate provides a much higher negative effect in the fault distance estimate than in the fault resistance one [19].

2.3. Distribution systems laterals

Power distribution feeders are typically radial networks composed by a main feeder, laterals, and sub-laterals. The faulted branch identification is a research field still under development, which exceeds the scope of this paper. The proposed fault analysis algorithm considers the faulted lateral to be previously known data. The extension of the proposed fault resistance sub-routine for systems with laterals, however, is still necessary. To obtain such extension, the proposed method obtains equivalent networks associated with each one of the system’s laterals.

In this work, each system lateral is replaced by an equivalent network that is represented as a three-phase constant impedance matrix. Thus, each system lateral can be represented by a tapped off equivalent network. Considering a distribution system with \(n\) laterals, \(n\) different equivalent networks are obtained, one for each lateral. These equivalents are obtained considering the systems pre-fault steady state operating conditions.

The lateral equivalents are calculated through a technique based on [20]. It uses estimated voltages and currents at each bus, considering several different systems conditions. The load impedance, however, must be maintained constant during the analysis. Hence, in this work, loads are represented as constant impedances. As opposed to [20], three-phase power flow analyses (PFA) are used to determine the equivalents. By the use of this approach, all equivalent networks are determined, and the system is reduced to the main path between the substation and the faulted section’s downstream bus. The steps to obtain such equivalent networks are detailed in the following:

(I) Three different power flow analyses, such as described in [20], are executed. In each one of the PFA, consider different voltage conditions at the substation, in order to obtain different operating points of the system. The system loading must be held constant in the three PFA. The different operating points are given by the different voltage conditions at the substation terminals. Initially is considered a balanced voltage condition at the substation terminals, as given by (16):

\[V_{S} = [V_{S} \angle 0^\circ, V_{S} \angle -120^\circ, V_{S} \angle 120^\circ]^T\]

The first PFA considers the balanced voltage condition, as given by (16). The other two PFA are executed considering slightly different voltage conditions, which should be unbalanced in order to avoid linearly dependent results. The voltage conditions considered in the subsequent PFA can be described by (17):

\[V_{S} = |V_{S}| \cdot (1 + \beta_k)\]

where \(\beta_k\) is a voltage deviation per unit value, which can be either a positive or a negative value. Its value should be small, such as 0.01 pu., and different for each phase \(k\). This avoids convergence problems in the power flow algorithm. The voltage angles, however, are kept constant.

(II) Calculate the equivalent impedances for each phase, from each node \(p\) to its adjacent nodes \(q\), using (18):

\[Z_{eq}|_{pq} = \begin{bmatrix}
Z_{eq} & Z_{eb} & Z_{ec} \\
Z_{eb} & Z_{bb} & Z_{bc} \\
Z_{ec} & Z_{bc} & Z_{cc}
\end{bmatrix} \cdot
\begin{bmatrix}
I_{eq} \\
I_{eb} \\
I_{ec}
\end{bmatrix}
\]

(18)

for \(t = \{a, b, c\}\), where the subscripts \(\{1, 2, 3\}\) represent the index of the three different PFA; and \(p, q\) is the adjacent nodes; \(Z_{mn}\) the impedance between phases \(m\) and \(n\); \(V_t\) the resulting voltage from PFA; \(I_t\) is the resulting current from PFA.

(II) Determine a three-phase equivalent matrix, \([Z_{eq}]_{pq}\), for each lateral that starts with a section connecting the adjacent nodes \(p\) and \(q\), using the vectors defined on (18), applied in (19):

\[Z_{eq}|_{pq} = |Z_{eq}|_{pq} \cdot |Z_{eq}|_{pq}^T\]

(19)

The algorithm considers initially the voltages and currents measured at the substation terminal. With the previously known information of the faulted section and the equivalent laterals, it is possible to calculate the voltages and currents at all nodes upstream to the faulted section.

Since voltages and currents are measured only at the substation terminals, voltages and currents at the downstream bus (\(k + 1\)) are estimated by (20) and (21), respectively:

\[V_{k+1} = |V_{k} - |Z_{k}| \cdot |I_{k}|\]

(20)

\[I_{k+1} = \sum_{m=1}^{n} |I_{k-m} - |Z_{k}| \cdot |I_{k}|\]

(21)

where \([Z_{k}]\) is the line impedance matrix between buses \(k\) and \(k + 1\); \([I_{k}]\) the three-phase current vector between buses \(k + 1\); \([I_{k-m}]\) the three-phase current vector between buses \(k\) and \(m\); \(n\) is the total system bus number.

Considering a constant impedance load model, the local bus load current \((I_{k})\) is calculated through (22):

\[I_{k} = |V_{k} \cdot |Z_{k}|^{-1}\]

(22)

where \([Z_{k}]\) is the bus \(k\) load impedance matrix.

The voltages and currents are updated until the first node upstream to the faulted section. These values are used for the fault resistance estimation procedure previously described.
2.4. Fault type extension

Sections 2.1–2.3 presented a fault resistance sub-routine for SLG faults. The technique may be extended to all remaining faults types through specific equations. For each fault type, the fault resistance sub-routine remains the same as presented previously. However, the fault resistance and distance equations (8) and (9), respectively, are replaced by the specific expressions for each fault type, as follows.

2.4.1. Line-to-line fault

Consider a line-to-line fault (L–L), as illustrated in Fig. 2. Following the same procedure presented in Section 2.1, a new set of expressions is obtained for fault location and resistance, given in a matrix form by (23):

\[
\begin{bmatrix}
  x \\
  R_f
\end{bmatrix} =
\begin{bmatrix}
  M_3 & I_{Fp0} \\
  M_4 & I_{Fp0}
\end{bmatrix}^{-1}
\begin{bmatrix}
  V_{Sfp0} - V_{Sfp0} \\
  V_{Sfp0} - V_{Sfp0}
\end{bmatrix}
\]

(23)

where

\[
M_3 = \sum_{k=a,b,c} [(z_{pk0} - z_{pk0})I_{gk0} - (z_{pk0} - z_{pk0})I_{gk0}]
\]

(24)

\[
M_4 = \sum_{k=a,b,c} [(z_{pk0} - z_{pk0})I_{gk0} + (z_{pk0} - z_{pk0})I_{gk0}]
\]

(25)

\[z_{pm} \text{ is the mutual impedance between phases } p \text{ and } m \text{ (} \Omega/\text{m}); z_{qm} \text{ the mutual impedance between phases } q \text{ and } m \text{ (} \Omega/\text{m}); m \text{ the phases } a, b, \text{ and } c; p, q \text{ is the faulted phases } a, b, \text{ or } c.\]

2.4.2. Double line-to-ground fault

A double line-to-ground fault (DLG) is illustrated in Fig. 3. The mathematical expressions for this fault type are given by:

\[
\begin{bmatrix}
  x \\
  R_{Fp} \\
  R_{Fq} \\
  R_{Fg}
\end{bmatrix} =
\begin{bmatrix}
  M_{1,2} & I_{Fp0} & 0 & 0 \\
  M_{2,2} & I_{Fp0} & 0 & I_{Fq0} + I_{Fg0} \\
  M_{1,4} & 0 & I_{Fp0} & I_{Fp0} + I_{Fq0} + I_{Fg0} \\
  M_{4,4} & 0 & 0 & I_{Fp0} + I_{Fq0} + I_{Fg0}
\end{bmatrix}^{-1}
\begin{bmatrix}
  V_{Sfp0} \\
  V_{Sfp0} \\
  V_{Sfp0} \\
  V_{Sfp0}
\end{bmatrix}
\]

(26)

where \(M_1\) and \(M_2\) are given by (5) and (6).

2.4.3. Three-phase fault

For the three-phase fault (3PH), which is illustrated in Fig. 4, the expressions for the fault distance and the three fault resistances are given by (27):

\[
\begin{bmatrix}
  x \\
  R_{Fe} \\
  R_{Fb} \\
  R_{Fg}
\end{bmatrix} =
\begin{bmatrix}
  M_{1,3} & I_{Fp0} & 0 & 0 \\
  M_{2,3} & I_{Fp0} & 0 & 0 \\
  M_{1,4} & 0 & I_{Fp0} & 0 \\
  M_{4,4} & 0 & 0 & I_{Fp0}
\end{bmatrix}^{-1}
\begin{bmatrix}
  V_{Sfp0} \\
  V_{Sfp0} \\
  V_{Sfp0} \\
  V_{Sfp0}
\end{bmatrix}
\]

(27)

where \(M_1\) and \(M_2\) are given by (5) and (6).

Finally, from (9), (23), (26), and (27) it is possible to estimate all fault resistances for any fault type. Based on these estimated values, the bus impedance sub-routine can be developed, as presented in the following.

3. Bus impedance sub-routine

Power distribution systems are typically unbalanced, with untransposed feeders and single-phase loads. In these conditions, the bus impedance matrix technique is the most suitable choice available for fault system voltages and currents state estimation. Using this method it is possible to analyze any fault type by modifying the phase coordinate base-case impedance matrix, considering the system’s asymmetries. The bus impedance sub-routine, which was initially proposed in [7], is described in the following subsections.

3.1. Bus impedance matrix

Three-phase admittance matrix \(Y_{bus}\) is calculated from the three-phase sub-matrices feeder’s components. The diagonal sub-matrix of a hypothetical bus \(p\) is calculated through (28), which represents the sum of all sub-matrices representing the \(M\) elements adjacent to bus \(p\).

\[
[Y_{abc}]_{pp} = \sum_{i=1}^{M} [Y_{abc}]_{ip}
\]

(28)

The off-diagonal sub-matrices are obtained from (29), where \([Y_{abc}]_{ij}\) is the admittance matrix element between buses \(p\) and \(j\).

\[
[Y_{abc}]_{ij} = -[Y_{abc}]_{ji}
\]

(29)
From (28) and (29), \(3n \times 3n\) three-phase admittance and impedance matrices are built, as presented in (30) and (31), respectively.

\[
[Y_{bus}] = \begin{bmatrix} Y_{abc,1,1} & Y_{abc,1,2} & \cdots & Y_{abc,1,n} \\ Y_{abc,2,1} & Y_{abc,2,2} & \cdots & Y_{abc,2,n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{abc,n,1} & Y_{abc,n,2} & \cdots & Y_{abc,n,n} \end{bmatrix}
\]  
(30)

\[
[Z_{bus}] = [Y_{bus}]^{-1}
\]  
(31)

where \(n\) is the total system bus number.

### 3.2. Pre-fault state calculation

The bus impedance sub-routine uses a ladder based three-phase load flow technique [21], considering the non-linear characteristics of the feeder to estimate the pre-fault voltages on each bus. For the estimated voltages convergence analysis, the three-phase load flow also considers the pre-fault voltages measured at the substation on each fault record and compares to the calculated voltages at the substation bus.

#### 3.3. Fault calculation

The bus impedance sub-routine is based on the superposition technique. The fault condition is simulated by two voltage sources series connected. The first voltage source represents the pre-fault voltage, whereas the second source is defined to satisfy each fault type boundary condition.

For each fault type, a different bus impedance matrix \([Z_{bus}]\) modification must be developed to include the fault resistance estimate, calculated by the fault resistance algorithm presented in Section 2. The \([Z_{bus}]\) modifications for each fault type are presented in the following.

##### 3.3.1. Single line-to-ground fault

For a single line-to-ground fault, \([Z_{bus}]\) must be modified to include the fault resistance of the path between the faulted line and ground. The fault resistance is included in the bus impedance matrix through a fault node \(r\). As result, a new impedance matrix \([Z_{new}]\) of dimension \((3n + 3) \times (3n + 3)\) is obtained. For a SLG fault, the fault current can be calculated using (32):

\[
I_{F} = \frac{V_{F}}{Z_{new}(r, r)}
\]  
(32)

where \(V_{F}\) is the node \(r\) pre-fault voltage, and

\[
Z_{new}(r, r) = Z_{bus}(r, r) + R_{F}
\]  
(33)

where \(R_{F}\) is the fault resistance, estimated by (9).

The superimposed voltages at each bus due to the injected fault current \(I_{F}\) may be calculated using the impedance matrix \([Z_{new}]\):

\[
\begin{bmatrix} \Delta V_{1} \\ \vdots \\ \Delta V_{r} \\ \vdots \\ \Delta V_{n} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} I_{F} \\ \vdots \\ -I_{F} \\ \vdots \\ 0 \end{bmatrix}
\]  
(34)

where (34) can also be rewritten as:

\[
\Delta V_{i} = Z_{new}(i, r) \cdot (-I_{F})
\]  
(35)

whereas \(i = 1 \rightarrow (3n + 3)\) represents the node index.

Adding the superimposed voltages to each pre-fault bus voltage \(V_{P}\), the fault period bus voltages \(V_{F}\) are estimated through (36):

\[
V_{F} = V_{P} + \Delta V_{i}
\]  
(36)

Finally, from the three-phase bus voltages and the admittance matrix, it is possible to calculate the three-phase currents during the fault period at each feeder \(I_{gh}\):

\[
I_{gh} = [Y_{abc}]_{gh} \cdot ([V_{F}] - [V_{P}])
\]  
(37)

where \([Y_{abc}]_{gh}\) is the admittance matrix between buses \(g\) and \(h\); \([V_{F}]\) the bus \(g\) three-phase voltage vector; \([V_{P}]\) is the bus \(h\) three-phase voltage vector.

where \(g, h = 1 \rightarrow (n + 1)\), in which \(n\) is the total bus number.

##### 3.3.2. Line-to-line fault

A similar procedure may be applied for line-to-line faults, where \([Z_{bus}]\) must be modified to include the fault resistance between the two faulted nodes \((r, s)\), as given by (38) and (39)

\[
Z_{new}(r, r) = Z_{bus}(r, r) + 0.5R_{F}
\]  
(38)

\[
Z_{new}(s, s) = Z_{bus}(s, s) + 0.5R_{F}
\]  
(39)

where \(R_{F}\) is the fault resistance estimated by (9).

From the modified impedance matrix, it is possible to determine the relation between the fault voltages and currents:

\[
\begin{bmatrix} V_{F1} \\ V_{F2} \end{bmatrix} = \begin{bmatrix} Z_{new}(r, r) & Z_{new}(r, s) \\ Z_{new}(s, r) & Z_{new}(s, s) \end{bmatrix} \begin{bmatrix} I_{F} \\ I_{P} \end{bmatrix}
\]  
(40)

Manipulating algebraically (40), the fault current for a line-to-line fault can be calculated using (41):

\[
I_{F} = \frac{V_{PP} - V_{PF}}{Z_{new}(r, r) - Z_{new}(r, s) + Z_{new}(s, s)}
\]  
(41)

where \(V_{PP}\) and \(V_{PF}\) are the nodes \(r\) and \(s\) pre-fault voltages.

With the estimated fault current, the superimposed voltages are calculated though (42):

\[
\begin{bmatrix} \Delta V_{1} \\ \vdots \\ \Delta V_{r} \\ \vdots \\ \Delta V_{n} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ -I_{F} \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} I_{F} \\ \vdots \\ -I_{F} \\ \vdots \\ 0 \end{bmatrix}
\]  
(42)

or

\[
\Delta V_{i} = \sum_{j=1}^{3n+3} Z_{new}(i, j) \cdot I_{j}
\]  
(43)

where \(i, j = 1 \rightarrow (3n + 3)\) represent the node indexes.

From the superimposed and pre-fault voltages, the three-phase voltages and currents at each feeder can also be calculated through (36) and (37), respectively.

##### 3.3.3. Double line-to-ground fault

Considering a DLG fault, as illustrated in Fig. 3, the \([Z_{bus}]\) matrix must be modified to include in the two faulted nodes \((r, s)\) the three fault resistances of the fault model. Following the same procedure as presented previously:

\[
Z_{new}(r, r) = Z_{bus}(r, r) + R_{F} + 0.5R_{Fs}
\]  
(44)

\[
Z_{new}(s, s) = Z_{bus}(s, s) + R_{F} + 0.5R_{Fs}
\]  
(45)

where \(R_{F}, R_{Fs}\), and \(R_{Ff}\) are the fault resistances estimates obtained from (28).

Using the modified three-phase impedance matrix \([Z_{new}]\), the fault currents \((I_{F, r}\) and \(I_{F, s})\) are calculated through (46):

\[
\begin{bmatrix} \Delta V_{1} \\ \vdots \\ \Delta V_{r} \\ \vdots \\ \Delta V_{n} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ -I_{F, r} \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} I_{F, r} \\ \vdots \\ -I_{F, s} \\ \vdots \\ 0 \end{bmatrix}
\]  
(46)
where \( V_{PFr} \), \( V_{PFs} \), and \( V_{PFT} \) are the pre-fault voltages on the faulted nodes \( r \) and \( s \), respectively. From the fault currents estimated through (46), the superimposed voltages can be obtained through (47):

\[
\begin{bmatrix}
\Delta V_r \\
\vdots \\
\Delta V_s \\
\vdots \\
\Delta V_t \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\end{bmatrix} \begin{bmatrix}
-V_{PFr} \\
\vdots \\
-V_{PFs} \\
\vdots \\
-V_{PFT} \\
\end{bmatrix}.
\]

From the superimposed voltages, the fault period bus voltages and currents can be obtained through (36) and (37), respectively.

### 3.3.4. Three-phase fault

Three-phase fault is the most severe fault type, and is normally used for rotating machines fault analysis. However, this fault type represents only 5% of the fault events on transmission lines [1]. For three-phase faults the bus impedance matrix must be modified to include the phases \( a, b, \) and \( c \) fault resistances, whose estimates are obtained from (27):

\[
\begin{align*}
Z_{new}(r, r) &= Z_{bus}(r, r) + R_{F}^r \\
Z_{new}(s, s) &= Z_{bus}(s, s) + R_{F}^s \\
Z_{new}(t, t) &= Z_{bus}(t, t) + R_{F}^t
\end{align*}
\]

where \( r, s, \) and \( t \) are the fault nodes indexes. From the modified three-phase bus impedance \( Z_{new} \), the fault currents are calculated using (51):

\[
\begin{bmatrix}
I_r \\
\vdots \\
I_s \\
\vdots \\
I_t \\
\end{bmatrix} = \begin{bmatrix}
-Z_{new}(r, r) & Z_{new}(r, s) & Z_{new}(r, t) \\
Z_{new}(s, r) & -Z_{new}(s, s) & Z_{new}(s, t) \\
Z_{new}(t, r) & Z_{new}(t, s) & -Z_{new}(t, t) \\
\end{bmatrix}^{-1} \begin{bmatrix}
V_{PFr} \\
\vdots \\
V_{PFs} \\
\vdots \\
V_{PFT} \\
\end{bmatrix}
\]

where \( V_{PF} \), \( V_{PFs} \), and \( V_{PFT} \) are the pre-fault voltages on the faulted nodes \( r, s, \) and \( t \), respectively.

From the fault currents calculated through (51), the bus superimposed voltages are obtained by (52):

\[
\begin{bmatrix}
\Delta V_r \\
\vdots \\
\Delta V_s \\
\vdots \\
\Delta V_t \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{Z_{new}} & \frac{1}{Z_{new}} & \ldots & \frac{1}{Z_{new}} \\
\end{bmatrix} \begin{bmatrix}
-V_{PFr} \\
\vdots \\
-V_{PFs} \\
\vdots \\
-V_{PFT} \\
\end{bmatrix}.
\]

Finally, as presented previously, the bus voltages and currents during the fault period are calculated by (36) and (37).

### 4. Case study

In order to validate the proposed algorithm, a modified version of IEEE 37 Node Test Feeder has been chosen for numeric test simulations. The original IEEE 37-bus system [16] has been modified to include some Brazilian’s distribution systems typical characteristics:

- Base voltage: 13.8 kV.
- Three wire Y-connected with neutral grounding.
- No voltage regulator.

- All loads modeled as constant impedance Y-connected with neutral grounding.

The modified test system, composed by 36 buses and illustrated by Fig. 5, was simulated using BPA’s ATP/EMTP [17]. The fault analysis algorithm was implemented in MATLAB® [15]. A modified Fourier filter [22] was also implemented in MATLAB to remove the decaying DC component and estimate the voltages and currents fundamental components measured at the substation terminal. The simulations were executed considering the following fault conditions:

- 103 different fault locations (covering all system laterals and sections).
- Five different fault resistances: 0, 10, 20, 50, and 100 Ω.
- 10 fault types: A-g, B-g, C-g, AB, BC, AC, AB-g, BC-g, AC-g, and ABC-g.
- Total: 5150 faults.

The simulation set was divided in two different case tests, which are summarized in the following scenarios:

#### Set I:

- 103 fault locations.
- Five fault resistances.
- Total: 5150 fault cases.

#### Set II:

- Two fault locations \((K_1 = 2.063 \text{ km}, K_2 = 1.649 \text{ km})\).
- Five fault resistances.
- 10 fault types.
- Total: 100 fault cases.

Fig. 5. Modified IEEE 37 node test feeder.
The errors in the fault resistances estimates were calculated considering the absolute difference between estimated and simulated fault resistances, as given by (53):

\[ e = |R_{\text{calc}} - R_{\text{real}}| \quad (53) \]

where \( R_{\text{calc}} \) and \( R_{\text{real}} \) are the estimated and the real fault resistances values, respectively.

5. Results

In the following subsections, the obtained results from the proposed fault analysis algorithms are presented and discussed. Initially, the effects of different fault resistance and distance values on the fault resistance estimate are verified. After, the buses voltages and fault currents estimates are also compared to the BPA’s ATP/EMTP simulations.

5.1. Fault resistance estimation

Set I test results were used to analyze the fault analysis algorithm performance. The results are analyzed over two aspects: fault resistance and distance effects.

5.1.1. Fault resistance effects

Table 1 presents the results for single line-to-ground and line-to-line faults for five simulated fault resistances. The results show that the errors associated to the fault resistances estimates slightly increase for higher fault resistances values. The algorithm yielded a highest average error equal to 1.13 \( \Omega \) for SLG faults and 0.19 \( \Omega \) for L-L faults. Still, the maximum errors produced by the algorithm were 1.66 \( \Omega \) and 1.13 \( \Omega \) for single line-to-ground and line-to-line faults, respectively.

Table 2 shows the fault resistances estimate results for double line-to-ground and three-phase faults. The comparison between the calculated and the simulated fault resistances values also demonstrate negligible errors on both fault types. In double line-to-ground faults, the average and maximum errors obtained were, respectively, 0.97 \( \Omega \) and 1.43 \( \Omega \), both occurred on the 100-\( \Omega \) fault scenario. The algorithm provided for three-phase faults a maximum error equal to 2.30 \( \Omega \), for a 100-\( \Omega \) fault. Also for this fault type, the highest average errors for 50-\( \Omega \) and 100-\( \Omega \) fault resistances were 0.22 \( \Omega \) and 1.05 \( \Omega \), respectively.

The results presented in Table 2 show slight differences between the estimated fault resistances in each faulted phase, resulted by system unbalances and the fundamental components determination process, due to different fault inception angles. However, as described in Table 2, these inaccuracies provide small differences which will not affect the proposed fault analysis method performance.

As expected from an impedance-based formulation, the highest errors were obtained during the most critical fault resistance test scenario. The fault resistance effect in the proposed algorithm may be explained by the erroneous estimation of the fault current for high fault resistances values [16], also associated to the so-called reactance error [23]. As given in (10), the proposed formulation is fault period load current estimate dependent. For faults with small fault resistances values, the current divider circuit of the faulted system is composed by the load impedance and a negligible fault resistance. In this scenario, the source current will mainly feed the fault and the fault current will be close to the first. Therefore, small variation on the calculated fault current does not affect the fault resistance estimate. As illustrated by Tables 1 and 2, as the fault resistance increases, this effect became more significant. However, even to the most critical analyzed fault scenario, the maximum error obtained was 2.30%.

5.1.2. Fault distance effect

To analyze the fault distance effect on the fault resistance estimate, the obtained results from Set I were used. Figs. 6 and 7 illustrate the fault resistance estimates over each one of the 103 simulated fault locations for single line-to-ground and line-to-line faults, respectively.

The fault resistance estimates demonstrate that the proposed algorithm efficiency is fault location independent. As provided by Tables 1 and 2, on faults up to 100-\( \Omega \), a small difference between maximum and average errors can be observed for the four fault types. Also, as illustrated by Figs. 6 and 7, a small variation over the 103 fault points can only be visually observed for the most critical simulated fault condition. However, as discussed previously, these differences can be neglected.

5.2. System voltages and currents estimation

Considering Set test II, the proposed fault analysis algorithm performance is discussed over two aspects: fault current at the faulty bus and bus voltages during the disturbance. In this section two different fault locations have been analyzed. The investigated fault points \( K_1 \) and \( K_2 \), which are coincident to buses 707 and 737, are located 2063 and 1649 meters from the substation, as illustrated in Fig. 5. The results discussed in this section represents not only the inaccuracy associated to the 2bus analysis technique, but the hole fault analysis method proposed in this paper, including the errors introduced by the fault resistance estimates.

5.2.1. Fault current

Fault analysis performance was first evaluated considering the fault current at faulty buses \( K_1 \) and \( K_2 \). Tables 3 and 4 show the fault current errors, demonstrating the inaccuracies of absolute and angular components between calculated fault current and BPA’s ATP/EMTP simulations.

The obtained results for single line-to-ground and line-to-line faults at \( K_1 \) and \( K_2 \) are presented in Table 3. During SLG faults without fault resistance, the method provided maximum errors to this

Table 1
Fault resistance estimates for single line-to-ground and line-to-line faults.

<table>
<thead>
<tr>
<th>Fault type</th>
<th>Simulated ( R_x ) (( \Omega ))</th>
<th>Maximum error (( \Omega ))</th>
<th>Minimum error (( \Omega ))</th>
<th>Average error (( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-g</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A-g</td>
<td>10</td>
<td>0.02</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>A-g</td>
<td>20</td>
<td>0.06</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>A-g</td>
<td>50</td>
<td>0.35</td>
<td>0.03</td>
<td>0.21</td>
</tr>
<tr>
<td>A-g</td>
<td>100</td>
<td>1.66</td>
<td>0.11</td>
<td>1.13</td>
</tr>
<tr>
<td>BC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BC</td>
<td>10</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BC</td>
<td>20</td>
<td>0.03</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BC</td>
<td>50</td>
<td>0.11</td>
<td>0</td>
<td>0.03</td>
</tr>
<tr>
<td>BC</td>
<td>100</td>
<td>1.13</td>
<td>0</td>
<td>0.19</td>
</tr>
</tbody>
</table>
fault type equal to 0.22% at $K_1$ and 0.30% at $K_2$, which represents negligible absolute errors of 16 and 21 A, respectively. Still in this fault scenario, the highest fault current angle errors were 0.31° and 0.16°. In line-to-line disturbances, a similar behavior as SLG faults has also been obtained, providing maximum errors of 0.54%, or 39 A, and 0.17°, both occurred at $K_2$. For 2LG and 3PH faults, which results at $K_1$ are shown in Table 4, the proposed fault analysis confirms its accuracy with small errors for all fault conditions. During double line-to-ground faults, the maximum error was 0.47% and 0.41°, once again occurred during solid faults. For three-phase faults the method also achieved negligible errors, equal to 0.59% and 0.37°.

The above mentioned results demonstrate that the proposed fault analysis method has always produced the highest absolute errors during solid faults, oppositely as the fault resistance estimate process, as discussed previously. However, the highest obtained error was 43 A, or 0.59%, and could be neglected. As shown by Tables 3 and 4, the inaccuracy associated to the absolute fault currents became insignificant for higher fault resistances values. Therefore, the fault resistance increase does not interfere in the fault current estimate accuracy.

### Table 2
Fault resistances estimates for double line-to-ground and three-phase faults.

<table>
<thead>
<tr>
<th>Fault type</th>
<th>Simulated $R_f$ (Ω)</th>
<th>Maximum error (Ω)</th>
<th>Minimum error (Ω)</th>
<th>Average error (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_a$</td>
<td>$R_b$</td>
<td>$R_c$</td>
<td>$R_a$</td>
</tr>
<tr>
<td>AC-g</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AC-g</td>
<td>10</td>
<td>–</td>
<td>10</td>
<td>0.01</td>
</tr>
<tr>
<td>AC-g</td>
<td>20</td>
<td>–</td>
<td>20</td>
<td>0.04</td>
</tr>
<tr>
<td>AC-g</td>
<td>50</td>
<td>–</td>
<td>50</td>
<td>0.29</td>
</tr>
<tr>
<td>AC-g</td>
<td>100</td>
<td>–</td>
<td>100</td>
<td>1.43</td>
</tr>
<tr>
<td>ABC-g</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ABC-g</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0.02</td>
</tr>
<tr>
<td>ABC-g</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>0.42</td>
</tr>
<tr>
<td>ABC-g</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>1.53</td>
</tr>
</tbody>
</table>

### Table 3
Fault current errors at $K_1$ and $K_2$ for single line-to-ground and line-line faults.

<table>
<thead>
<tr>
<th>Fault</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{fa}$ (%)</td>
<td>$\Theta$ (°)</td>
</tr>
<tr>
<td>AC-g</td>
<td>0.22</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Fig. 6. Distance effect on $R_f$ estimation for SLG faults (A-g).

Fig. 7. Distance effect on $R_f$ estimation for L–L faults (BC).
5.2.2. Bus voltages

The fault analysis algorithm performance was also evaluated through comparisons between BPA’s ATP/EMTP simulations and the estimated voltages in four different buses: 702, 709, 720, and 734. Table 5 presents the maximum absolute and angular errors for all analyzed faults at $K_1$. Considering the bus voltages estimated from the calculated fault current and pre-fault voltages, this section discusses the inaccuracies introduced by three different processes: fault resistance, fault current and three-phase load flow.

The estimated bus voltages accuracy is demonstrated by Table 5 results: the highest absolute error was 0.82%, which represents an error equal to 113 Volts and it is associated to bus 720. A maximum error of 0.72% has also been obtained at bus 734. As consequence from the already discussed fault current estimate accuracy, the results provided by Table 5 also demonstrate the bus voltage estimate invariance to different fault resistances values. Besides, these results demonstrate that the main error source is the pre-fault bus voltages, which were estimated in this paper through a three-phase load flow based on the ladder technique [21]. Therefore, the bus voltage estimates are dependent to the bus and also the fault location, resulting in different error levels (absolute and angular) to each analyzed bus. However, even the highest obtained error may be neglected due its small value.

### References


### Table 4

Fault current errors at $K_1$ for double line-to-ground and three-phase faults.

<table>
<thead>
<tr>
<th>Fault type</th>
<th>$R_f$ (Ω)</th>
<th>$I_{fA}$ (%)</th>
<th>$\Theta_A$ (°)</th>
<th>$I_{fB}$ (%)</th>
<th>$\Theta_B$ (°)</th>
<th>$I_{fC}$ (%)</th>
<th>$\Theta_C$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>0</td>
<td>0.21</td>
<td>0.41</td>
<td>0.47</td>
<td>0.40</td>
<td>0.27</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.01</td>
<td>0.35</td>
<td>0.06</td>
<td>0.38</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.01</td>
<td>0.35</td>
<td>0.03</td>
<td>0.37</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0</td>
<td>0.34</td>
<td>0.01</td>
<td>0.37</td>
<td>0</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.01</td>
<td>0.34</td>
<td>0.01</td>
<td>0.37</td>
<td>0.01</td>
<td>0.34</td>
</tr>
</tbody>
</table>

### Table 5

Bus voltages errors for faults at $K_1$.

<table>
<thead>
<tr>
<th>$R_f$ (Ω)</th>
<th>Fault type</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V$ (%)</td>
<td>Θ (%)</td>
<td>$V$ (%)</td>
<td>Θ (%)</td>
<td>$V$ (%)</td>
<td>Θ (%)</td>
</tr>
<tr>
<td>702</td>
<td>A-g</td>
<td>0.25</td>
<td>0.2</td>
<td>0.25</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.21</td>
<td>0.17</td>
<td>0.24</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>AC-g</td>
<td>0.20</td>
<td>0.16</td>
<td>0.24</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>0.18</td>
<td>0.14</td>
<td>0.24</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>709</td>
<td>A-g</td>
<td>0.17</td>
<td>0.31</td>
<td>0.18</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.12</td>
<td>0.31</td>
<td>0.17</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>AC-g</td>
<td>0.21</td>
<td>0.30</td>
<td>0.19</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>0.11</td>
<td>0.31</td>
<td>0.17</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td>720</td>
<td>A-g</td>
<td>0.71</td>
<td>0.50</td>
<td>0.82</td>
<td>0.51</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.75</td>
<td>0.64</td>
<td>0.81</td>
<td>0.51</td>
<td>0.82</td>
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<tr>
<td></td>
<td>AC-g</td>
<td>0.77</td>
<td>0.48</td>
<td>0.82</td>
<td>0.50</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>0.76</td>
<td>0.51</td>
<td>0.82</td>
<td>0.50</td>
<td>0.82</td>
</tr>
<tr>
<td>734</td>
<td>A-g</td>
<td>0.37</td>
<td>0.43</td>
<td>0.40</td>
<td>0.39</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.15</td>
<td>0.72</td>
<td>0.37</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>AC-g</td>
<td>0.18</td>
<td>0.47</td>
<td>0.37</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>0.20</td>
<td>0.62</td>
<td>0.38</td>
<td>0.39</td>
<td>0.39</td>
</tr>
</tbody>
</table>

### 6. Conclusions

In this paper, an iterative fault analysis algorithm is proposed. The proposed method is based on two instances: fault resistance estimation process, through an iterative impedance-based formulation; and the fault voltages and currents estimation process, which are carried out through a bus impedance matrix based formulation. The proposed technique aims to help post-disturbance analysis, providing information to Distribution Operation Centers. The proposed technique has been developed to be suitable for generic unbalanced power distribution systems with ramifications, and only requires the disturbance sending-end voltages and currents.

Test results demonstrate an accurate and robust fault analysis method. The fault resistance estimate algorithm provided accurate results to all analyzed cases, for faults up to 100 Ω, with a small dependence with the fault resistance value. Besides, the fault resistance sub-routine is invariant to the fault location. The proposed fault analysis algorithm performance is also independent to fault resistance values and fault location. The obtained results demonstrate the technique accuracy to all analyzed conditions, showing that the discussed aspects do not affect the proposed fault analysis performance.

In this work the fault impedance was modeled as a constant resistance and the distribution system considered radial. It is expected that the proposed method performance decreases when fault impedance is not strictly resistive and constant or when distributed generation are connected to network buses. Parameter identification is a research topic of great interest of the power systems community. In this work however this issue was not addressed. The proposed formulation considers known accurate system data. It is expected that the proposed method performance decreases with data inaccuracy.

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