Further improvements on impedance-based fault location for power distribution systems

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Abstract: In this study, further improvements regarding the fault location problem for power distribution systems are presented. The proposed improvements relate to the capacitive effect consideration on impedance-based fault location methods, by considering an exact line segment model for the distribution line. The proposed developments, which consist of a new formulation for the fault location problem and a new algorithm that considers the line shunt admittance matrix, are presented. The proposed equations are developed for any fault type and result in one single equation for all ground fault types, and another equation for line-to-line faults. Results obtained with the proposed improvements are presented. Also, in order to compare the improvements performance and demonstrate how the line shunt admittance affects the state-of-the-art impedance-based fault location methodologies for distribution systems, the results obtained with two other existing methods are presented. Comparative results show that, in overhead distribution systems with laterals and intermediate loads, the line shunt admittance can significantly affect the state-of-the-art methodologies response, whereas in this case the proposed developments present great improvements by considering this effect.

1 Introduction

Electric power systems are constantly exposed to faults, which affect the system’s reliability, security and power quality. Different stochastic events may cause system faults, such as lightning, insulation breakdown and trees falling across lines. Protection schemes are important to maintain system stability and minimise consumer and network damages, as well as economical losses. In these aspects, fault location (FL) techniques represent an important role in the fast and reliable power system restoration process.

FL in electric power distribution systems (EPDS), however, because of their specific topological and operational characteristics, still presents challenges. Still today, FL in EPDS is often performed through visual inspection, field methods and brute force methods [1]. These techniques are not feasible on underground systems and require a long time in large distribution networks.

Impedance-based FL techniques for EPDS are especially attractive because of their low implementation cost (particularly one-terminal-based ones) in relation to high-frequency techniques, such as those using travelling waves and wavelet analysis. Since the 1980s, FL techniques for EPDS have been increasingly developed, specially those based on the apparent impedance. In the beginning of this development, power system modelling was performed considering the lines as geometrically symmetrical by using a symmetrical-components-based analysis [2–6]. Symmetrical components-based methodologies were also developed specifically to underground distribution systems, accounting the line shunt admittance (LSA), as proposed in [7]. Such methodologies are of great importance nowadays, in spite of their limitation to balanced systems.

Recently, several FL methods using the phase components approach have been proposed [8–13]. These methods consider together the EPDS main characteristics (unbalanced operation, presence of intermediate loads, laterals/sublaterals and time-varying load profile). Despite recent suggestions that those methods are not the most appropriate in relation to the symmetrical-components-based ones [14], they actually comprise the state-of-the-art impedance-based FL methods for EPDS. The tests presented in [14] consider a distribution feeder with transposed lines, which is not the most general type of line configuration and does not yield the greatest advantages of the phase components approach.

None of the cited methods, symmetrical components based or phase components based, however, did consider the capacitive effect of the distribution lines (i.e. its shunt admittance) together with its inherent unbalance. Actually, very few impedance-based FL techniques did consider such effect in EPDS [15, Chapter 8]. This consideration is in part by the low shunt admittance values of distribution lines in comparison with transmission lines ones, especially on overhead distribution systems, since underground ones can achieve higher capacitance values because of the distribution cables’ high proximity [16]. However, as will be shown in this work that, the LSA can significantly affect the impedance-based FL methods for EPDS response, even for overhead distribution lines. This effect was considered
for FL purposes in EPDS only in [17], where an iterative algorithm was used to account for the capacitive current in underground systems, thus leading to an iterative algorithm inside an iterative algorithm. The present paper shows that it is possible to obtain FL equations that directly account for the LSA in a typical iterative FL algorithm.

Based on these considerations, the present work aims at three main objectives:

1. To include the LSA on general one-terminal impedance-based FL equations for EPDS, considering all shunt fault types;
2. To develop and present an algorithm for FL on general EPDS, considering its laterals and intermediate loads, using only local measurements;
3. To evaluate the LSA effects on distribution systems FL, considering the developed improvements and the state-of-the-art FL techniques.

The remaining of this paper is organised as follows. Section 2 presents the proposed FL equations for ground and phase faults. The modified FL algorithm for systems with intermediate loads is presented in Section 3, whereas its generalisation for systems with laterals is presented in Section 4. Section 5 presents the studied fault cases, as well as the studied system. The results obtained with the studied methods when the capacitive effect was neglected and considered in the simulations are presented in Sections 6 and 7, respectively. In Section 8 these results are compared and discussed. The conclusions are presented in Section 9.

2 Proposed generalised FL equations

First consider a distribution line represented through its exact line segment model, as shown in Fig. 1. Using Kirchoff’s voltage and current laws, it is possible to write that [16]

\[
\begin{bmatrix}
    V_{abc}
    \\
    I_{abc}
\end{bmatrix}
= 
\begin{bmatrix}
    d_e & -b_e & a_e
    \\
    -c_e
\end{bmatrix}
\begin{bmatrix}
    V_{abc}
    \\
    I_{abc}
\end{bmatrix}
\tag{1}
\]

where

\[
a_e = d_e + 0.5 \cdot \ell^2 \cdot Z_{abc} \cdot Y_{abc}
\tag{2}
\]

\[
b_e = \ell \cdot Z_{abc}
\tag{3}
\]

\[
c_e = \ell \cdot Y_{abc} + 0.25 \cdot \ell^3 \cdot Z_{abc} \cdot Y_{abc}
\tag{4}
\]

and

\[
V_{abc} \quad \text{three-phase (3PH) terminal m voltages vector (in volts)};
\]

\[
I_{abc} \quad \text{3PH terminal m currents vector (in amps)};
\]

\[
I \quad \text{third-order identity matrix};
\]

\[
f \quad \text{total line length (in km)};
\]

\[
Z_{abc} \quad \text{line series impedance matrix (in ohms/km)};
\]

\[
Y_{abc} \quad \text{line shunt admittance matrix (in ohms}^{-1} \text{/km)}.
\]

For a fault located \( x \) kilometres from the beginning of the line, (1) can be rewritten as

\[
V_F = d_e \cdot V_S - b_e \cdot I_S
\tag{5}
\]

where \( d_e \) and \( b_e \) are defined through (2) and (3), respectively, replacing \( f \) by \( x \), and \( V_F \) denotes the fault point voltages vector.

It is clear from (5) that the LSA interferes in the FL as observed from the terminal voltages and currents through the term \( d_e \), which includes \( Y_{abc} \). The proposed generalised FL equations can be developed from this equation, as described in the following subsections for ground faults and phase faults, respectively. A brief discussion on the relation between the proposed equations and the currently existing ones is presented on Appendix.

2.1 Ground faults

Consider Fig. 2, which presents the most general ground fault model, since it represents single line-to-ground (SLG), double line-to-ground (DLG), and three phase-to-ground faults [18]. The voltage/current relation at the fault point for this model is given by

\[
\begin{bmatrix}
    V_{F_a}
    \\
    V_{F_b}
    \\
    V_{F_c}
\end{bmatrix}
= 
\begin{bmatrix}
    Z_{F_a} + Z_{g} & Z_{F_b} & Z_{F_c}
    \\
    Z_{F_b} & Z_{F_a} + Z_{g} & Z_{F_c}
    \\
    Z_{F_c} & Z_{F_b} & Z_{F_a} + Z_{g}
\end{bmatrix}
\begin{bmatrix}
    I_{F_a}
    \\
    I_{F_b}
    \\
    I_{F_c}
\end{bmatrix}
\tag{6}
\]

where \( V_{F_a} \), fault point voltage on phase \( k \) (in volts); \( I_{F_a} \), fault current on phase \( k \) (in amps); \( a, b, c \), phases related to the variables; \( Z_{F_a+c} \), fault impedances (in ohms), as defined in Fig. 2.

In (6), only the faulted phases have fault currents that are different from zero (\( I_{F_i} \neq 0 \)). Replacing (6) in (5) it is possible to write for each faulted phase \( k \) that

\[
Z_{F_k} \cdot I_{F_k} + Z_{g} \cdot I_F = V_{S_k} + x^2 / 0.5 \cdot M_k - x \cdot N_k
\tag{7}
\]
where $I_F$ comprises the sum of fault currents in all faulted phases, as shown in Fig. 2 and

\[
\begin{bmatrix}
M_a \\
M_b \\
M_c
\end{bmatrix} = \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \cdot \begin{bmatrix}
Y_{aa} & Y_{ab} & Y_{ac} \\
Y_{ba} & Y_{bb} & Y_{bc} \\
Y_{ca} & Y_{cb} & Y_{cc}
\end{bmatrix} \begin{bmatrix}
V_{Sa} \\
V_{Sb} \\
V_{Sc}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
N_a \\
N_b \\
N_c
\end{bmatrix} = \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \cdot \begin{bmatrix}
I_{Sa} \\
I_{Sb} \\
I_{Sc}
\end{bmatrix}
\]

It should be noted that (7) results in $n$ equations, where $n$ represents the total number of faulted phases.

Splitting (7) into its real and imaginary parts and considering $Z_{Fk}^a$, $Z_{Fk}^b$ and $Z_{Fk}^c$ as pure resistances results, respectively, in

\[
R_{Fk} I_{Fk}^a + R_{Fk} I_{Fk}^b - X_{Fk} I_{Fk}^c = V_{Sk}^a + x^2 \cdot 0.5 \cdot M_k - x \cdot N_k = T_{kr}
\]

and

\[
R_{Fk} I_{Fk}^a + R_{Fk} I_{Fk}^b + X_{Fk} I_{Fk}^c = V_{Sk}^b + x^2 \cdot 0.5 \cdot M_k - x \cdot N_k = T_{ki}
\]

where the subscripts $r$ and $i$ represent the real and imaginary parts of the variables, respectively.

As a result, $2n$ equations and $2n$ unknowns are obtained, considering that the fault impedances are also unknowns besides the fault distance, $x$. Using (10) and (11), it is possible to isolate the fault resistance in each faulted phase, $R_{Fk}^a$, and match the equations. Thus, a set of $n$ equations are obtained which are independent from $R_{Fk}^a$ and dependent from $x$, $R_{Fk}^a$, and $X_{Fk}^a$. The equations are given by

\[
R_{Fk} = \frac{1}{I_{Fk}^a} \cdot [T_{kr} - R_{Fk} I_{Fk}^a + X_{Fk} I_{Fk}^c]
\]

\[
= \frac{1}{I_{Fk}^a} \cdot [T_{ki} - R_{Fk} I_{Fk}^a - X_{Fk} I_{Fk}^c]
\]

Rewriting (12) it is possible to obtain for each faulted phase $k$ that

\[
\sum_{k \in \Omega_k} \{R_{Fk} \cdot \Im \{I_{Fk}^a \cdot I_{Fk}^b \} - X_{Fk} \cdot \Real \{I_{Fk}^a \cdot I_{Fk}^c \} + [T_{kr} - R_{Fk} \cdot I_{Fk}^a + X_{Fk} \cdot I_{Fk}^c] = 0
\]

(13)

Multiplying (13) by $(-I_{Fk}^a \cdot I_{Fk}^b)$ and conducting algebraic manipulations related to complex algebra, it is possible to obtain

\[
R_{Fk} \cdot \Im \{I_{Fk}^a \cdot I_{Fk}^b \} - X_{Fk} \cdot \Real \{I_{Fk}^a \cdot I_{Fk}^c \} + [T_{kr} - R_{Fk} \cdot I_{Fk}^a - X_{Fk} \cdot I_{Fk}^c] = 0
\]

(14)

where $\Real \{\cdot \}$ and $\Im \{\cdot \}$ represent the real and imaginary parts of complex numbers, respectively, and $\ast$ denotes a complex conjugate.

For each faulted phase $k$, (14) can be rewritten, depending on the number of phases subjected to fault. Summing these $n$ equations it is possible to write one single equation, given by

\[
R_{Fk} \cdot \Im \{I_{Fk}^a \cdot I_{Fk}^b \} - X_{Fk} \cdot \Real \{I_{Fk}^a \cdot I_{Fk}^c \} + \sum_{k \in \Omega_k} [T_{kr} - R_{Fk} \cdot I_{Fk}^a - X_{Fk} \cdot I_{Fk}^c] = 0
\]

(15)

With knowledge that

\[
\Im \{I_{Fk}^a \cdot I_{Fk}^b \} = \Im \{I_{Fk}^a \} = 0
\]

and considering a strictly resistive fault ($X_{Fk} = 0$), the following sum is obtained

\[
\sum_{k \in \Omega_k} [T_{kr} - R_{Fk} \cdot I_{Fk}^a - X_{Fk} \cdot I_{Fk}^c] = 0
\]

(17)

where $\Omega_k$ is the set of faulted phases, given by a combination of phases from the system $(a, b$ and $c)$. In a 3PH system there are seven possible combinations for ground faults, involving one, two or three phases.

Replacing $T_{kr}$ and $T_{ki}$ from (10) and (11) in (17) and conducting algebraic manipulations involving complex algebra, the FL equation for ground faults can be finally written as

\[
x^2 \cdot \left[ 0.5 \cdot \sum_{k \in \Omega_k} \Im \{M_k \cdot I_{Fk}^b \} \right] - x \cdot \left[ \sum_{k \in \Omega_k} \Im \{N_k \cdot I_{Fk}^c \} \right] + \left[ \sum_{k \in \Omega_k} \Im \{V_{Sk}^a \cdot I_{Fk}^a \} \right] = 0
\]

(18)

which is called the ‘Generalised Ground-FL Equation’, or GFLE.

It should be noted that the GFLE is a FL equation in which it is possible to estimate the fault distance by using the 3PH voltages and currents measured at the substation, the line parameters (series impedance and shunt admittance), and also the fault current. These variables are used to calculate the coefficients of (18), considering (8) and (9). Since the fault current is also unknown from the local terminal measurements, a formulation for estimating its value should be used, as subsequently presented in Section 3.

### 2.2 Phase faults

Considering a line-to-line (LL) fault, as illustrated in Fig. 3, it is possible to obtain from (5), (8) and (9)

\[
I_{Fk} = -I_{Fk}^a
\]

\[
V_{Fk} = V_{Sk}^a + x^2 \cdot 0.5 \cdot M_k - x \cdot N_k
\]

\[
= \frac{V_{Sk}^a + x^2 \cdot 0.5 \cdot M_k - x \cdot N_k + Z_F \cdot I_{Fk}}{V_{Sk}^a}
\]

(19)

(20)
Isolating $RF$ in (21) and replacing it into (22) yields

$$x^2 \cdot 0.5 \cdot \left[ \frac{M_a - M_b}{I_{F_a}} - \frac{M_a - M_b}{I_{F_a}} \right] - x \cdot \left[ \frac{N_a - N_b}{I_{F_a}} - \frac{N_a - N_b}{I_{F_a}} \right] + \left[ \frac{V_{S_a} - V_{S_b}}{I_{F_a}} - \frac{V_{S_a} - V_{S_b}}{I_{F_a}} \right] = 0$$

(23)

Multiplying (23) by $(I_{F_a} \cdot I_{F_a})$, the final equation results in (24)

$$x^2 \cdot 0.5 \cdot 3 \{ (M_a - M_b) \cdot I_{F_a}^2 \} - x \cdot 3 \{ (N_a - N_b) \cdot I_{F_a}^2 \} + 3 \{ (V_{S_a} - V_{S_b}) \cdot I_{F_a}^2 \} = 0$$

(24)

Generalising (24) for LL faults involving any $i$ and $k$ phases results in

$$x^2 \cdot 0.5 \cdot 3 \{ (M_i - M_k) \cdot I_{F_i}^2 \} - x \cdot 3 \{ (N_i - N_k) \cdot I_{F_i}^2 \} + 3 \{ (V_{S_i} - V_{S_k}) \cdot I_{F_i}^2 \} = 0$$

(25)

which is called the ‘Generalised Phase-FL Equation’ or PFLE.

As the GFLE, the PFLE is an FL equation in which it is possible to estimate the fault distance by using the 3PH voltages and currents at the substation, the line parameters, and also the fault current. These variables are used to calculate the coefficients of (25), considering (8) and (9). The same way used for estimating the fault current of the GFLE can be used on the PFLE.

### 3 Modified FL algorithm for systems with intermediate loads

The current existing FL algorithms for EPDS neglect the LSA and consider the distribution line as an RL circuit [8, 9, 12, 14]. Also, as presented in Section 2, the proposed FL equations are actually second-order polynomials in $x$, the fault distance to the relay location. This situation was not anticipated by the existing FL algorithms. Thus, these algorithms are not fully appropriate for use with the proposed FL equations. Hence, the algorithm proposed in [9] was modified and its usage is proposed to solve the FL problem by using the GFLE and the PFLE, proposed in this paper. First, only the intermediate loads was taken into account, and the existence of laterals/sublaterals is considered through an extended algorithm subsequently detailed in Section 4. The modified algorithm is described in detail in the following:

**Algorithm 1:**

1. Determine an initial fault current estimate, using (26)

$$I_F = I_{S_k} - I_S$$

(26)

where $I_{S_k}$ and $I_S$ are, respectively, the vector representing the currents measured at the relay location during the fault and before fault occurrence;

2. Determine the solutions of the GFLE or PFLE, given, respectively, by (18) and (25);

3. Determine the physically correct solution referring to the fault, as subsequently described in Section 3.1;

4. Check if $x$ has converged by calculating

$$|x(n) - x(n - 1)| < \delta$$

(27)

for $n > 1$, where $\delta$ is a predefined tolerance, given in [km], and $n$ is the iteration number;

5. If $x$ converged for the analysed section or converged for the last section, return $x$. If $x$ converged for a location beyond the current Section, update $V_S$ and $I_S$ to the next node of the system using (1) (i.e. change the reference node) and go back to step 1. Otherwise, go to step 6;

6. Calculate the voltages at the fault point, $V_S$, using (5), $x(n)$, and the currents and voltages at the node right upstream to the analysed section ($V_{S_k}$ and $I_{S_k}$, where $k$ is the reference node);

7. Update the current that is downstream to the fault, in the faulted phases, $I_D$, using the calculated fault point voltages, according to (28)

$$I_D = [Z_{total}^{-1} + 0.5 \cdot (\ell - x) \cdot Y_{abc}] \cdot V_F$$

(28)

where

$$Z_{total} = (\ell - x) \cdot Z_{abc} + [0.5 \cdot (\ell - x) \cdot Y_{abc} + Z_{k+1}^{-1}]^{-1}$$

(29)

and $Z_{k+1}$ is the impedance connected to node $k + 1$;

8. Update the fault current, according to (30)

$$I_F = I_U - I_D$$

(30)
where \( I_L = -c_e \cdot V_S + a_e \cdot I_S \) is the current upstream to the fault point, according to (1);

9. Go back to step II.

### 3.1 Determining the physically correct solution

The proposed FL equations are second-order polynomials in \( x \), the fault distance. Thus, in each iteration of the previously described algorithm, two new fault distances are calculated. Only one of the solutions, however, corresponds to the physical location of the fault. The other calculated distance is purely mathematical and does not have a physical meaning. The determination of which solution is physically correct leads to a comprehensive discussion, and shall be suppressed in this paper because of space limitations. A detailed discussion is presented in [19]. The conclusion is given in the following.

Considering that a quadratic polynomial can be represented as

\[
\alpha_2 \cdot x^2 + \alpha_1 \cdot x + \alpha_0 = 0
\]

the fault distance, \( x \), calculated through the GLFE of the PFLE that represents the physically correct solution, is given by

\[
x = \begin{cases} 
-\alpha_1 + \sqrt{\alpha_1^2 - 4 \cdot \alpha_2 \cdot \alpha_0} \div 2 \cdot \alpha_2, & \text{if } \alpha_1 > 0 \\
-\alpha_1 - \sqrt{\alpha_1^2 - 4 \cdot \alpha_2 \cdot \alpha_0} \div 2 \cdot \alpha_2, & \text{if } \alpha_1 < 0
\end{cases}
\]

(32)

Also, as described in [19], one should consider that complex solutions might arise from the GFLE and the PFLE because of modelling, measurement or estimation errors. Meanwhile, as long as these errors are kept within acceptable limits, the solutions will only be complex for a small range of fault-resistance values. In these situations, the fault distance is determined as the absolute value of the solutions.

### 4 Proposed generalised algorithm for systems with laterals and intermediate loads

Algorithm 2 generalises Algorithm 1 by accounting laterals and sublaterals. It is explained in the following:

#### Algorithm 2:

1. Determine the node equivalents, as subsequently explained in Section 4.1;
2. Determine the possible paths and their equivalents, as subsequently explained in Section 4.2;
3. Start from the first path;
4. Run Algorithm 1 for the current path;
5. Save the calculated FL to \( x(p) \), where \( p \) is the path number;
6. If the analysed path is the last one \( (p = p_{\text{max}}) \), go to step 7. Otherwise, update \( p (p = p + 1) \) and go back to step 4;
7. Determine the correct estimated FL, using a proper existing algorithm, such as [8, 9, 20].

It is well known that for general power distribution systems with laterals, one-terminal-based impedance-based FL techniques yield several FL estimates, because of the tree nature topology of such systems. In this paper, no specific technique for determining which is the correct one is proposed. Hence, step 7 presents not only a suggestion, but also a need, of the usage of a proper existing algorithm for such task. Several of them are already suitable for systems with LSA considered in its modelling. It is also possible to determine the correct fault section by using other techniques and devices, such as fault indicators. The idea in this paper, however, is to present just the FL technique and not the fault section estimation. Thus, no further considerations and/or analysis on this subject will be presented.

#### 4.1 Determination of node equivalents

The consideration of equivalent impedances to deal with distribution systems with laterals has already been proposed in previously published papers [9, 12]. The idea used in the proposed approach is strongly based on [12], because of its great advantage on using 3PH power-flow analysis to determine these equivalents. The main improvements done to the methodology proposed in [12] is the consideration of the LSA in the power-flow analysis (which is straightforward) and the calculation of equivalents with phase coupling included. In order to obtain these equivalents, the method proposed in [21] was adapted to the studied FL problem. The full procedure is presented as follows:

**Algorithm 3:**

1. Define three hypothetical operating points for the system, where the load and the system topology remain constant, and the 3PH voltage setpoints (magnitude and/or angle) defined at the substation are different;
2. Calculate the power flow for each one of the chosen hypothetical operating points, to obtain the voltages and currents in each node and section of the system;
3. Using the results of step 2, define matrix \( V_{tm} \) for each line section \( t \) of the system using (33) (following the notation of Fig. 1)

\[
V_{tm} = \begin{bmatrix} V_{tm_1} & V_{tm_2} & V_{tm_3} \\ V_{tm_2} & V_{tm_3} & V_{tm_4} \\ V_{tm_3} & V_{tm_4} & V_{tm_5} \end{bmatrix}
\]

(33)

where \( V_{tm_m} \) stands for the voltage of node \( m \) of section \( t \), phase \( a' \), obtained with the power-flow analysis under operating point number \( \beta \). Accordingly, also define matrices \( V_m, I_m \) and \( I_m^1 \);
4. Calculate two equivalent impedances for each section \( t \) of the system using (34) and (35)

\[
Z_{eq_m} = I_m^{-1} \cdot V_m
\]

(34)

\[
Z_{eq_m} = I_m^{-1} \cdot V_m
\]

(35)

It should be noted that step 2 of Algorithm 3 requires some previous inspection to obtain three operating points where the power-flow analysis converges. The experience of the authors is that if a backward/forward sweep algorithm is used, then a maximum change in 0.1 p.u. in voltage magnitude and 0.2 rad in angle are sufficient for the proposed analysis and results in convergence of the power flow. Also, the voltage setpoint changes should be in only one phase for each different operating point, in order to provide three linearly independent solutions. This is not an usual operating point, for the voltages at the substation would be unbalanced. This is why they are called
hypothetical operating points, as they are just a mathematical tool used to obtain the 3PH equivalents. The usage of any three different operating points should lead to the same equivalents (in practice).

As can be noticed, Algorithm 3 was developed for 3PH systems. If any section of an analysed system has less than three phases, then the matrices $V_{tm}, V_{tn}, I_{tm}$ and $I_{tn}$ are defined including only the existing phases. Also, the number of power-flow solutions used to define these matrices is the same as the total number of phases.

The calculated impedances are used in two different algorithms. Matrix $Z_{eqtm}$ is used in the path equivalents, as explained subsequently in Section 4.2, whereas $Z_{eqtn}$ is used directly into Algorithm 1 as $Z_{k+1}$, the equivalent impedance downstream to the analysed section.

### 4.2 Determination of possible paths and its equivalents

To consider each lateral, the proposed method calculates equivalent systems to each possible path, resulting into $n$ equivalent radial systems, where $n$ is the total number of laterals, which are suitable to be analysed by algorithm 1. The equivalent systems are obtained by the transformation of lines and loads outside the path being analysed into constant impedances along the radial system. The algorithm used is proposed in [12] and is transcribed below:

**Algorithm 4:**
1. Determine the $n$ possible paths;
2. Select one path and determine the nodes with laterals;
3. For each node with lateral, determine an equivalent load, considering only the previously calculated equivalent impedances, $Z_{eqtm}$, outside the path being analysed;
4. For each node with lateral and also loads, do the parallel between loads and the equivalent loads determined on step 3. This is the final equivalent load for the nodes in the path being analysed;
5. Go back to step 2 until all the $n$ equivalent systems are determined.

### 5 Studied cases

In order to evaluate how does the LSA affect the state-of-the-art impedance-based FL techniques for EPDS, a numerical analysis was carried out considering several fault cases. The state-of-the-art methods that were evaluated in this paper are Lee’s *et al.* method, with its extensions [9, 12], and Choi’s *et al.* method [10, 11]. These fault cases were simulated under BPA’s ATP-EMTP software [22] in an overhead power distribution system. The test system is a modified version of IEEE 34 Node Test Feeder [23]. The cited system had its lines changed to only one configuration (3PH, configuration number 300), and its distributed loads were aggregated as spot loads, as shown in Table 1. Line lengths were kept unchanged. Also, the loads were considered to be 3PH and its voltage regulators were removed for simplification purposes. Line sections were modelled as cascaded $\pi$-circuits, with the number of $\pi$-circuits representing each section depending on the length of each one. Actually, two versions of this system were modelled in ATP and studied: one with its LSA neglected in its ATP model, and other one considering it.

In each simulated fault, the 3PH voltages and currents at the local terminal were measured. The studied fault cases in each modeled system are summarised in Table 2. Each studied methodology was tested for each simulated fault,
with the exception of Choi’s method [10, 11], which was developed only for SLG and LL faults. The idea of the chosen simulations sets is to compare the results obtained in a system without LSA (Case I) with a system with LSA (Case II). This will demonstrate that the LSA significantly affects the results of the state-of-the-art FL methodologies and that the proposed improvements help decreasing the LSA impacts.

Since voltages and currents obtained through BPA’s ATP-EMTP are in the time domain, a modified Fourier filter was used to estimate the phasors [24], representing the systems fault state in phasor domain. This filter and the studied FL techniques were implemented under Matlab [25]. The performance of all methods was evaluated through the fault distance estimate, \( x_{est} \), percentage error, given by

\[
\text{error[\%]} = 100 \frac{|x_{sim}[\text{km}] - x_{est}[\text{km}]|}{\ell_{line}[\text{km}]}
\]  

where \( \ell_{line} \) represents the total line length and \( x_{sim} \) is the simulated fault distance. The analysed system has a total line length of 92.18 km, which is the sum of the lengths of all line sections.

It should be noticed that since the proposed improvements do not include a method for determining the faulted lateral (its methodology may yield several different ones), the faulted lateral determination was not analysed during the tests. Thus, the correct faulted lateral was always correctly chosen (since we already know the faulted lateral), and the FL estimate error was analysed. This also applies to the other analysed methods.

6 Results obtained neglecting the capacitive effect

The results presented in this section are the ones related to Case I. Lee and Choi’s methods were used to locate faults in a system with line shunt admittance neglected in its ATP models. The following subsections discuss some aspects regarding the obtained results.

6.1 Fault resistance effect

Results obtained with the faults comprised in Case I are presented in Tables 3 and 4 as a function of the fault resistance, for Lee and Choi’s methods. Each row of the table aggregates results for four or five different fault resistances, considering faults in all line sections. It is possible to observe that as the fault resistance, \( R_F \), increases, the error also increases for all fault types.

It is important to notice, however, that even with high \( R_F \) values (100 \( \Omega \)) the average errors were always very small, below 0.35% for both methods. The maximum errors were also very low, below 0.75% for both methodologies. This observation agrees with previously obtained results [9–11], and it is also lower than the results presented in the cited papers, which shows the robustness of these methodologies. One should be aware that these results were obtained while neglecting the LSA during the simulations, which is in

### Table 3 Percentage errors as a function of the fault resistance for Lee et al.’s method [9, 12] – Case I

<table>
<thead>
<tr>
<th>( R_F ), ( \Omega )</th>
<th>A-g</th>
<th>B-g</th>
<th>C-g</th>
<th>AB-g</th>
<th>BC-g</th>
<th>AC-g</th>
<th>AB</th>
<th>BC</th>
<th>AC</th>
<th>ABC-g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average error, %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–15</td>
<td>0.23</td>
<td>0.23</td>
<td>0.24</td>
<td>0.24</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>20–35</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>40–60</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>65–80</td>
<td>0.26</td>
<td>0.26</td>
<td>0.27</td>
<td>0.27</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>85–100</td>
<td>0.27</td>
<td>0.28</td>
<td>0.29</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>Maximum error, %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–15</td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
<td>0.36</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>20–35</td>
<td>0.39</td>
<td>0.38</td>
<td>0.38</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>40–60</td>
<td>0.42</td>
<td>0.42</td>
<td>0.41</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.39</td>
</tr>
<tr>
<td>65–80</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.44</td>
<td>0.45</td>
<td>0.44</td>
<td>0.44</td>
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<td>0.44</td>
</tr>
<tr>
<td>85–100</td>
<td>0.49</td>
<td>0.50</td>
<td>0.51</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.49</td>
<td>0.49</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### Table 4 Percentage errors as a function of the fault resistance for Choi et al.’s method [10, 11] – Case I

<table>
<thead>
<tr>
<th>( R_F ), ( \Omega )</th>
<th>Average error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–15</td>
<td>0.23</td>
</tr>
<tr>
<td>20–35</td>
<td>0.24</td>
</tr>
<tr>
<td>40–60</td>
<td>0.26</td>
</tr>
<tr>
<td>65–80</td>
<td>0.29</td>
</tr>
<tr>
<td>85–100</td>
<td>0.32</td>
</tr>
<tr>
<td>Maximum error, %</td>
<td></td>
</tr>
<tr>
<td>0–15</td>
<td>0.37</td>
</tr>
<tr>
<td>20–35</td>
<td>0.41</td>
</tr>
<tr>
<td>40–60</td>
<td>0.51</td>
</tr>
<tr>
<td>65–80</td>
<td>0.60</td>
</tr>
<tr>
<td>85–100</td>
<td>0.71</td>
</tr>
</tbody>
</table>

accordance with the analysed FL methods, which also neglect the capacitive effect in their formulation.

6.2 Fault distance effect

Through the obtained results it is possible to observe that the fault distance effect on the analysed FL methods is less predictable than the fault-resistance effect. The estimation error increases abruptly for some fault distance values, as observed in previous analyses of these methods [9–11, 14]. These fault distances are exactly at the intersection between two different sections. For the same section, the error is almost the same, whereas from one section to another, the error sometimes is higher, and sometimes is lower. This observation leads to the conclusion that the fault distance does not significantly affect the FL estimate in the analysed methods. However, these abrupt error variations are a consequence of equivalent estimation errors. During the lateral equivalents calculation of these methods, some errors occur, resulting in the presented FL errors. The conclusion was the same for all analysed fault types.

6.3 Different fault types

Considering Tables 3 and 4, it is possible to observe that different fault types yield different percentage errors. Considering the four different fault types, that is, SLG, DLG, LL, and 3PH faults, it can be concluded that LL faults are the ones that yield the smallest percentage error, associated with the highest \( R_F \) values. This effect is more pronounced in Choi’s method. On the other hand, SLG faults were the ones that yielded the highest percentage errors under the same fault conditions. Thus, it is clear that the fault type affects the overall performance of the analysed method. This effect, however, does not seem to the be quite significant, because of the small difference between different fault types.

Regarding the same fault types, it is still possible to observe through Tables 3 and 4 a variation considering different phases subjected to fault. The most susceptible fault type to this effect was the LL fault. This difference is attributed to system unbalance (loads and line geometry) and also to phasor estimation errors, since there is no possible integration step given by a rational number that is a multiple of the fundamental frequency, 60 Hz, on ATP-EMTP. This fact occurs since the 60 Hz period is given by a rational number, \( 16.6666 \ldots \) ms, which when truncated returns a phasor estimation error. Since all faults were simulated to occur during the same simulation time, different fractions of each phase’s signal were erroneously considered/dropped.

7 Results obtained considering the capacitive effect

The results presented in this section are the ones related to Case II. Lee and Choi’s methods, together with the proposed improvements [12], were used to estimate FLs in a system with LSA considered in its ATP models. A similar analysis to the previous case is presented in the following subsections.

7.1 Fault-resistance effect

The results obtained with the faults comprised in Case II are presented in Tables 5–7 as a function of the fault resistance, \( R_F \), for Lee and Choi’s methods and the proposed improvements. It is possible to observe that as the fault resistance, \( R_F \), increases, the error also increases for all fault types, which is very similar to the results observed for Case I.

<table>
<thead>
<tr>
<th>( R_F, \Omega )</th>
<th>Average error, %</th>
<th>Maximum error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-g</td>
<td>B-g</td>
<td>C-g</td>
</tr>
<tr>
<td>0–15</td>
<td>0.40</td>
<td>0.46</td>
</tr>
<tr>
<td>20–35</td>
<td>0.97</td>
<td>1.07</td>
</tr>
<tr>
<td>40–60</td>
<td>2.11</td>
<td>2.20</td>
</tr>
<tr>
<td>65–80</td>
<td>3.73</td>
<td>3.77</td>
</tr>
<tr>
<td>85–100</td>
<td>5.60</td>
<td>5.54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average error, %</th>
<th>Maximum error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–15</td>
<td>0.89</td>
</tr>
<tr>
<td>20–35</td>
<td>1.87</td>
</tr>
<tr>
<td>40–60</td>
<td>3.69</td>
</tr>
<tr>
<td>65–80</td>
<td>5.66</td>
</tr>
<tr>
<td>85–100</td>
<td>8.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( R_F, \Omega )</th>
<th>Average error, %</th>
<th>Maximum error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-g</td>
<td>B-g</td>
<td>C-g</td>
</tr>
<tr>
<td>0–15</td>
<td>0.40</td>
<td>0.47</td>
</tr>
<tr>
<td>20–35</td>
<td>0.98</td>
<td>1.08</td>
</tr>
<tr>
<td>40–60</td>
<td>2.13</td>
<td>2.22</td>
</tr>
<tr>
<td>65–80</td>
<td>3.78</td>
<td>3.81</td>
</tr>
<tr>
<td>85–100</td>
<td>5.69</td>
<td>5.61</td>
</tr>
</tbody>
</table>
In this case, however, the average and maximum errors are substantially increased, when compared with the results obtained for Case I, for Lee and Choi’s methods. Note that the maximum errors shown in Tables 3 and 4 for high $R_F$ have a similar magnitude as the average ones found on Tables 5 and 6 for low $R_F$ values. The maximum errors on Case II were found to be at least nine times higher than the ones for Case I (Choi’s method, BC faults), and in some cases it increased more than 20 times (Lee’s method, ABC-g faults).

The proposed improvements also showed to be affected by the fault resistance, as shown in Table 7. This effect, however, showed to be less significant than in Lee and Choi’s methods. This conclusion is easily verified by comparing the maximum and average errors for all three methods, considering the faults with lowest and highest $R_F$ values. It can be observed that for low $R_F$ values, the proposed method shows errors that are very similar to Lee and Choi’s methods (two or even three times lower at most). As the fault resistance increases, however, the proposed method is able to maintain low error values, whereas the other two methods fail to maintain its good performance observed for low-impedance faults.

### 7.2 Fault distance effect

The same results presented in Tables 5–7 are presented in Figs. 4–6 for A-g faults with four different fault resistances, as a function of the fault distance, respectively, for Lee’s method, Choi’s method and the proposed improvements.

It is possible to see in this case that the equivalent estimate affected less significantly both Lee’s method and Choi’s method. Actually, in this case there are fewer abrupt variations. Also, the error presents a very uniform variation, almost maintaining its value for each fault resistance and is almost the same for both methods. This behaviour is the same for all other fault types.

The proposed improvements, however, showed to be strongly affected by fault distance, as illustrated in Fig. 6. Similar to the response of Lee and Choi’s methods in Case I, the proposed method suffers from abrupt variations. The proposed method, however, presents its lowest errors in the beginning of each line section, and the error rapidly increases with fault distance. When a new node is found, the errors go back again to lower levels. It should also be noted that the fault distance effect is pronounced for higher fault resistances, which shows the interaction between these two variables in the proposed method. Considering that the
studied system is actually a rural distribution feeder, with long lines (more than 90 km of lines), this effect should be of no concern for urban distribution feeders, which normally present significantly shorter lines. The same response is obtained for the other fault types, and thus, no figures are presented owing to space limitations.

### 7.3 Different fault types

Observing Tables 5 and 6, it is possible to observe that different fault types yield very different results for Lee and Choi’s methods in relation to their average and maximum errors. For Case I, this difference was not significant, although it was observable. The LSA, however, increased this effect, showing that for both methods, the LL fault type yielded the best results. The worst results were obtained for 3PH and SLG faults, for Lee and Choi’s methods, respectively.

Table 7 presented a lower discrepancy in the results for different fault types, when compared with Lee and Choi’s methods. However, although lower, the fault type affected the FL estimate, and the effects were similar to Lee’s, with lower errors for LL faults and higher errors for 3PH faults. Also, the very same fault distance effect with different intensities for each fault type can be observed.

### 8 Discussion

To present the significant improvements on FL performance by using the proposed developments, it is important to compare the obtained results using each FL method and the proposed developments. No new results are presented in this section, the previously presented ones are only compared with each other, considering only the results for faults on Case II, which are the only ones evaluated by all studied methods. The minimum, average and maximum errors are aggregated for all studied faults and are presented on Table 8.

As shown in Table 8, it is clear that the proposed developments presented significant improvements, in relation to average and maximum errors obtained. Regarding Lee et al.’s method, the average error was reduced in more than 3% in 3PH faults, a reduction of more than nine times. The maximum error was reduced in almost 9.5% for the same fault type, which means a reduction in about seven times of the maximum error. In comparison to Choi et al.’s method, the average error was reduced over 2% for B-g faults whereas the maximum error was reduced in almost 7% for the same fault type, which shows an improvement of the FL estimate in about 8.5 times and 6 times, respectively. As a rule to all studied FL methods, the smallest obtained errors are associated to LL faults, in all the analysed methodologies and system conditions, including the proposed improvements (maximum and average). In comparison with Lee et al.’s method, the average error was reduced in 1.3% for BC faults when using the proposed improvements, whereas the maximum error was reduced in almost 4% under the same fault conditions. This corresponds to a reduction of more than six times and five times in the FL estimate, respectively. Regarding Choi et al.’s method, this reduction was, respectively, of 0.9% and 3%, under the same fault conditions (reduction of more than four times for both). It is important to remember that 1% represents almost 1 km in the analysed system.

Fig. 7 presents comparative results, comprising the three analysed methodologies, as a function of the fault resistance, for A-g faults located at 13.1 km of the feeder. Through this figure the improvements for SLG fault types are clearly shown. Actually, for low $R_F$ values the results are not significantly different between the FL methods. As the fault resistance increases, however, the capacitive effect, which reinforces the $R_F$ effect on Lee et al.’s and Choi et al.’s methods, does not significantly affect the proposed improvements. This shows the actual improvements brought by the proposed method: it reduces the LSA effect reinforcement on the $R_F$ effect. Hence, it significantly reduces the FL error, even in overhead power distribution systems. The same effect is observed on the other fault types (DLG, LL and 3PH), and the figures are not included because of paper extension limits.
9 Conclusions

In this paper, a discussion on the LSA consideration for FL purposes on one-terminal impedance-based methods for power distribution systems was presented. The major contributions of this work are related to the presentation of the LSA effect on these existing FL methods. The obtained results show that even in overhead power distribution lines this effect should not be neglected since it can significantly increase the FL error in the existing methods. The capacitive effect consideration in the models used in electromagnetic simulations is a choice in the FL analysis. The capacitive effect, however, is always present in real distribution systems, even if the associated capacitances are extremely low (e.g. in overhead distribution systems).

This work also presented new impedance-based FL equations that do consider the LSA. Following the new equations, a modified FL algorithm was presented in which the LSA was also considered. Actually, the proposed FL equations are second-order polynomials in \( x \), the fault distance to the local terminal. The analysis of this equation was essential in order to understand the nature and response of its solutions under different fault conditions. Through this analysis it was also possible to determine which one of the two solutions is the physically correct and also, when complex solutions are to be expected.

It is important to notice that the studied fault cases were all associated with a real power distribution system with laterals and intermediate loads. Lee et al.’s [9, 12] and Choi et al.’s [10, 11] methods were analysed in the same system, under the same fault conditions, yielding significantly higher errors than when the LSA was neglected, showing the real effects of this variable. The comparative analysis elucidates that the proposed improvements performance is significantly superior to the cited methodologies performance in the analysed cases, suggesting promising evolutions on one-terminal impedance-based FL methods for power distribution systems. Ongoing research by the authors is focused on the sensitivity of the proposed developments with respect to measurement errors and parameters uncertainty. Future works should also present faulted lateral identification methods that consider the LSA, since this topic was not covered in this paper.

10 Acknowledgments

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11 References


12 Appendix

12.1 Relationship between the proposed equations and the existing ones

The proposed equations, GFLE and PFLE, given, respectively, by (18) and (25) can be easily modified to neglect the LSA, showing the relationship between the proposed equations with the current existing ones. In this case, the LSA matrix is neglected, which means that \( Y_{abc} = 0 \). This simplification results in \( M_{L} = 0 \), since \( M = Z_{abc} \cdot Y_{abc} \). In this case, the GFLE results in

\[
x = \frac{\sum_{k \in \mathbb{F}_{abc}} \left( V_{S_{k}} \cdot I_{P_{k}} \right)}{\sum_{k \in \mathbb{F}_{abc}} \left( V_{N_{k}} \cdot I_{P_{k}} \right)}
\]
for faults involving the phases in $\Omega_k$. Furthermore, the PFLE results in

$$x = \frac{\Im\{(V_{s_m} - V_{s_n}) \cdot I_{n_m}^* \}}{\Im\{(N_m - N_n) \cdot I_{n_m}^* \}}$$

(38)

for faults involving phases $m$ and $n$.

Equations (37) and (38) show that by neglecting the LSA, the second-order polynomials representing the proposed FL equations result in a linear equation in $x$, which means that both equations yield only one fault distance. Actually, it is possible to mathematically show that (37) and (38) are the same equations as proposed in [9, 12]. Only the 3PH fault case does not present a direct relationship, since for this fault type a RL fault impedance was considered in [12]. The existing equations actually correspond to special cases of the GFLE and the PFLE.

Also, by neglecting the LSA it is possible to use the already existing FL algorithms [8, 9, 12]. Thus, the proposed improvements can be thought as extensions of the current FL methods for EPDS, based on phase coordinates, to account for the capacitive effect on the distribution line.