Universidade Federal do Rio Grande do Sul
Escola de Engenharia
Departamento de Engenharia Elétrica

Sistemas e Sinais - Área II

FORMULÁRIO

Não rasurar. Entregar o formulário juntamente com a prova.
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I. SENOIDES COMPLEXAS E SISTEMAS LTI

A. Sinais discretos

$$x[n] = e^{j\Omega n}$$
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega(n-k)} = H(e^{j\Omega n})e^{j\Omega n}$$

B. Sinais contínuos

$$x(t) = e^{j\omega t}$$
$$y(t) = \int_{-\infty}^{\infty} h(\tau)e^{j\omega(t-\tau)} = H(j\omega)e^{j\omega t}$$

II. SÉRIES DE FOURIER

A. Sinais discretos

$$x[n] \overset{DTFS:\Omega_0}{\longleftrightarrow} X[k]$$

$$x[n] = \sum_{k=<N>} X[k]e^{jk\Omega_0 n}$$
$$X[k] = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n}, \quad \Omega_0 = \frac{2\pi}{N}$$

$x[n]$ e $X[k]$ têm período fundamental N

1) Pares básicos:

$$x[n] = \begin{cases} 1, & |n| \leq M \\ 0, & M < |n| \leq N/2 \end{cases}$$
$$X[k] = \frac{\sin\left[k\frac{\Omega_0}{2}(2M+1)\right]}{N\sin\left(k\frac{\Omega_0}{2}\right)}$$

B. Sinais contínuos

$$x(t) \overset{FS:\omega_0}{\longleftrightarrow} X[k]$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}$$
$$X[k] = \frac{1}{T}\int_{<T>} x(t)e^{-jk\omega_0 t}dt, \quad \omega_0 = \frac{2\pi}{T}$$

$x(t)$ tem período T

1) Pares básicos:

$$x(t) = \begin{cases} 1, & |t| \leq T_s \\ 0, & T_s < |t| \leq T/2 \end{cases}$$
$$X[k] = \frac{\sin(k\omega_0 T_s)}{k\pi}$$

III. TRANSFORMADAS DE FOURIER

A. Sinais discretos

$$x[n] \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega})$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$X(e^{j\Omega})$ tem período 2π

1) Pares básicos:

- $$x[n] = \begin{cases} 1, & |n| \leq M \\ 0, & \text{caso contrário} \end{cases}$$
  $$X(e^{j\Omega}) = \frac{\sin\left[\Omega\left(\frac{2M+1}{2}\right)\right]}{\sin\left(\frac{\Omega}{2}\right)}$$
- $$x[n] = \alpha^n u[n], \quad |\alpha| < 1$$
  $$X(e^{j\Omega}) = \frac{1}{1-\alpha e^{-j\Omega}}$$

B. Sinais contínuos

$$x(t) \overset{FT}{\longleftrightarrow} X(j\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

1) Pares básicos:

- $$x(t) = \begin{cases} 1, & |t| \leq T \\ 0, & \text{caso contrário} \end{cases}$$
  $$X(j\omega) = \frac{2\sin(\omega T)}{\omega}$$
- $$x(t) = e^{-at}u(t), \quad \Re\{a\} > 0$$
  $$X(j\omega) = \frac{1}{a+j\omega}$$

IV. PROPRIEDADES DAS REPRESENTAÇÕES DE FOURIER

A. Linearidade

$$\begin{aligned} z(t) &= ax(t)+by(t) \overset{FT}{\longleftrightarrow} Z(j\omega)=aX(j\omega)+bY(j\omega) \\ z(t) &= ax(t)+by(t) \overset{FS;\omega_0}{\longleftrightarrow} Z[k]=aX[k]+bY[k] \\ z[n] &= ax[n]+by[n] \overset{DTFT}{\longleftrightarrow} Z(e^{j\Omega})=aX(e^{j\Omega})+bY(e^{j\Omega}) \\ z[n] &= ax[n]+by[n] \overset{DTFS;\Omega_0}{\longleftrightarrow} Z[k]=aX[k]+bY[k] \end{aligned}$$

B. Deslocamento no tempo

$$\begin{aligned} x(t-t_0) &\overset{FT}{\longleftrightarrow} e^{-j\omega t_0}X(j\omega) \\ x(t-t_0) &\overset{FS;\omega_0}{\longleftrightarrow} e^{-jk\omega_0 t_0}X[k] \\ x[n-n_0] &\overset{DTFT}{\longleftrightarrow} e^{-j\Omega n_0}X(e^{j\Omega}) \\ x[n-n_0] &\overset{DTFS;\Omega_0}{\longleftrightarrow} e^{-jk\Omega_0 n_0}X[k] \end{aligned}$$

C. Deslocamento na frequência

$$\begin{aligned} e^{j\gamma t}x(t) &\overset{FT}{\longleftrightarrow} X(j(\omega-\gamma)) \\ e^{jk_0\omega_0 t}x(t) &\overset{FS;\omega_0}{\longleftrightarrow} X[k-k_0] \\ e^{j\Gamma n}x[n] &\overset{DTFT}{\longleftrightarrow} X(e^{j(\Omega-\Gamma)}) \\ e^{jk_0\Omega_0 n}x[n] &\overset{DTFS;\Omega_0}{\longleftrightarrow} X[k-k_0] \end{aligned}$$

D. Diferenciação, integração e somatório

$$\begin{aligned} \frac{d}{dt}x(t) &\overset{FT}{\longleftrightarrow} j\omega X(j\omega) \\ \frac{d}{dt}x(t) &\overset{FS;\omega_0}{\longleftrightarrow} jk\omega_0 X[k] \\ -jtx(t) &\overset{FT}{\longleftrightarrow} \frac{d}{d\omega}X(j\omega) \\ -jnx[n] &\overset{DTFT}{\longleftrightarrow} \frac{d}{d\Omega}X(e^{j\Omega}) \\ \int_{-\infty}^{t} x(\tau)d\tau &\overset{FT}{\longleftrightarrow} \frac{1}{j\omega}X(j\omega) \\ \sum_{k=-\infty}^{n} x[k] &\overset{DTFT}{\longleftrightarrow} \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} \end{aligned}$$

E. Convolução

$$\begin{aligned} x(t)*z(t) &\overset{FT}{\longleftrightarrow} X(j\omega)Z(j\omega) \\ x(t)\otimes z(t) &\overset{FS;\omega_0}{\longleftrightarrow} TX[k]Z[k] \\ x[n]*z[n] &\overset{DTFT}{\longleftrightarrow} X(e^{j\Omega})Z(e^{j\Omega}) \\ x[n]\otimes z[n] &\overset{DTFS;\Omega_0}{\longleftrightarrow} NX[k]Z[k] \end{aligned}$$

F. Modulação

$$\begin{aligned} x(t)z(t) &\overset{FT}{\longleftrightarrow} \frac{1}{2\pi}X(j\omega)*Z(j\omega) \\ x(t)z(t) &\overset{FS;\omega_0}{\longleftrightarrow} X[k]*Z[k] \\ x[n]z[n] &\overset{DTFT}{\longleftrightarrow} \frac{1}{2\pi}X(e^{j\Omega})\otimes Z(e^{j\Omega}) \\ x[n]z[n] &\overset{DTFS;\Omega_0}{\longleftrightarrow} X[k]\otimes Z[k] \end{aligned}$$

V. DIAGRAMA DE BODE

Sistema com zeros e polos reais, fase mínima

$$H(j\omega)=K\frac{\prod_{i=1}^{m}(j\omega+z_i)}{\prod_{j=1}^{n}(j\omega+p_j)}$$

$$\begin{aligned} |H(j\omega)| &= |K|\frac{\prod_{i=1}^{m}\sqrt{\omega^2+z_i^2}}{\prod_{j=1}^{n}\sqrt{\omega^2+p_j^2}} \\ \phi(H(j\omega)) &= \sum_{i=1}^{m}\arctan\left(\frac{\omega}{z_i}\right)-\sum_{j=1}^{n}\arctan\left(\frac{\omega}{p_j}\right) \end{aligned}$$

A. Classe 0 de termos

$$\begin{aligned} &H(j\omega)=\pm K \\ &20log(K) \\ &0^0 \text{ se } K>0,\ \ 180^0 \text{ se } K<0 \end{aligned}$$

B. Classe 1 de termos

$$\begin{aligned} &H(j\omega)=(j\omega)^{\pm\gamma} \\ &\pm\gamma 20db/dec \\ &\pm\gamma 90^0 \end{aligned}$$

C. Classe 2 de termos

$$H(j\omega)=\left(\frac{1/\tau}{j\omega+1/\tau}\right)^{\pm 1}=(j\omega\tau+1)^{\pm 1}$$

$$\begin{aligned} &\omega\tau << 1 \rightarrow 1;\ \ \omega\tau >> 1 \rightarrow \pm 20db/dec \\ &\omega\tau << 1 \rightarrow 0^0;\ \ \omega\tau >> 1 \rightarrow \pm 90^0, \\ &\omega\tau = 1 \rightarrow \pm 3db;\ \ \omega\tau = 1 \rightarrow \pm 45^0 \end{aligned}$$

D. Classe 3 de termos

$$H(j\omega)=\left[\frac{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2}{\omega_n^2}\right]^{\pm 1}=\left[\left(\frac{j\omega}{\omega_n}\right)^2+2\xi\frac{j\omega}{\omega_n}+1)\right]^{\pm 1}$$

$$\begin{aligned} &\omega << \omega_n \rightarrow 1;\ \ \omega >> \omega_n \rightarrow \pm 40db/dec \\ &\omega << \omega_n \rightarrow 0^0;\ \ \omega >> \omega_n \rightarrow \pm 180^0, \\ &\omega = \omega_n \rightarrow \pm 20log(2\xi);\ \ \omega = \omega_n \rightarrow \pm 90^0 \end{aligned}$$

VI. IDENTIDADES TRIGONOMÉTRICAS

$$\begin{aligned} &\cos^2(x)=\frac{1}{2}[1+\cos(2x)] \\ &\cos(a\pm b)=\cos(a)\cos(b)\mp\sin(a)\sin(b) \\ &\sin(a\pm b)=\sin(a)\cos(b)\pm\cos(a)\sin(b) \\ &\cos(a)+\cos(b)=2\cos(\frac{1}{2}(a+b))\cos(\frac{1}{2}(a-b)) \\ &\cos(a+b)+\cos(a-b)=2\cos(a)\cos(b) \\ &e^{\pm j\theta}=\cos(\theta)\pm j\sin(\theta) \\ &\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2} \\ &\sin(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j} \\ &\text{sinc}(u)=\frac{\sin(\pi u)}{\pi u} \end{aligned}$$

VII. RELAÇÕES TENSÃO/CORRENTE

$$\begin{aligned} &v_R(t)=Ri_R(t) \\ &\lambda_L(t)=Li_L(t)\Rightarrow v_L(t)=L\frac{d}{dt}i_L(t) \\ &q_C(t)=Cv_C(t)\Rightarrow v_C(t)=\frac{1}{C}\int_{-\infty}^{t}i_C(t)dt \end{aligned}$$

VIII. SÉRIE GEOMÉTRICA

Seja β um número complexo, então

$$\sum_{n=0}^{N-1}\beta^n=\begin{cases}\frac{1-\beta^N}{1-\beta}, & \beta\neq 1\\ N, & \beta=1\end{cases}$$

$$\sum_{n=0}^{\infty}\beta^n=\frac{1}{1-\beta},\qquad |\beta|<1$$