Universidade Federal do Rio Grande do Sul
Escola de Engenharia
Departamento de Engenharia Elétrica

Sistemas e Sinais - Área III

FORMULÁRIO

Não rasurar. Entregar o formulário juntamente com a prova.

I. AMOSTRAGEM

$$X_\delta(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

II. RECONSTRUÇÃO DE SINAIS

A. *Reconstrução ideal*

$$H_r(j\omega) = \begin{cases} T_s, |\omega| \le \omega_s/2 \\ 0, |\omega| > \omega_s/2 \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] sinc\left(\frac{\omega_s}{2\pi}(t - nT_s)\right)$$

B. *Reconstrução prática*

$$H_0(j\omega) = 2e^{-j\omega T_s/2}\frac{\sin\left(\omega\frac{T_s}{2}\right)}{\omega}$$

$$x_0(t) = \sum_{n=-\infty}^{\infty} x[n]h_0(t - nT_s)$$

C. *Filtro anti-imagem*

$$H_c(j\omega) = \begin{cases} \frac{\omega T_s}{2\sin(\omega T_s/2)}, |\omega| \le \omega_m \\ 0, |\omega| > \omega_s - \omega_m \end{cases}$$

III. TRANSFORMADA DE LAPLACE

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

- Teorema do valor inicial: $\lim_{s\to\infty} sX(s) = x(0^+)$
- Teorema do valor final: $\lim_{s\to 0} sX(s) = x(\infty)$

IV. TRANSFORMADA Z

$$X(z) = \sum_{k=-\infty}^{\infty} x(k)z^{-k}$$

V. FILTROS

A. *Filtros de Butterworth* $(1-\epsilon)^2 = 1-\gamma \qquad \delta^2 = \mu$

$H_{PB}(s) = \frac{\omega_c^N}{Q(s)}, \;\; H_{PA}(s) = \frac{s^N}{Q(s)}$ ver Tabela I

$$\omega_p = \omega_{cp}\left(\frac{2\epsilon - \epsilon^2}{(1-\epsilon)^2}\right)^{1/2N} \qquad \omega_s = \omega_{cs}\left(\frac{1-\delta^2}{\delta^2}\right)^{1/2N}$$

$$N = \frac{log\left(\frac{(2\epsilon-\epsilon^2)\delta^2}{(1-\epsilon)^2(1-\delta^2)}\right)}{2log\left(\frac{\omega_p}{\omega_s}\right)}$$

Sinal	Transformada de Laplace
$x(t-\tau)$	$e^{-s\tau}X(s)$
$e^{s_0 t}x(t)$	$X(s-s_0)$
$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^0\frac{d^{n-1}}{dt^{n-1}}x(t)\|_{t=0^-} - \ldots - s^{n-1}x(0^-)$
$-tx(t)$	$\frac{d}{ds}X(s)$
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}\int_{-\infty}^{0^+} x(\tau)d\tau + \frac{X(s)}{s}$
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$\delta(t-\tau),\ \tau > 0$	$e^{-s\tau}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\cos(\omega_1 t)u(t)$	$\frac{s}{s^2+\omega_1^2}$
$\sin(\omega_1 t)u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$

Sinal	Transformada $\mathcal{Z}$
$x[-n]$	$X(\frac{1}{z})$, RDC $\frac{1}{R_x}$
$x[n-k]$	$x[-k] + \ldots + z^{-k+1}x[-1] + z^{-k}X(z)$
$x[n+k]$	$-z^kx[0] - \ldots - zx[k-1] + z^kX(z)$
$nx[n]$	$-z\frac{d}{dz}X(z)$
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$, RDC $\|z\| > \alpha$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$, RDC $\|z\| < \alpha$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$
$\cos(\Omega_1 n)u[n]$	$\frac{1-z^{-1}\cos(\Omega_1)}{1-z^{-1}2\cos(\Omega_1)+z^{-2}}$
$\sin(\Omega_1 n)u[n]$	$\frac{z^{-1}\sin(\Omega_1)}{1-z^{-1}2\cos(\Omega_1)+z^{-2}}$
$r^n\cos(\Omega_1 n)u[n]$	$\frac{1-z^{-1}r\cos(\Omega_1)}{1-z^{-1}2r\cos(\Omega_1)+r^2z^{-2}}$
$r^n\sin(\Omega_1 n)u[n]$	$\frac{z^{-1}r\sin(\Omega_1)}{1-z^{-1}2r\cos(\Omega_1)+r^2z^{-2}}$

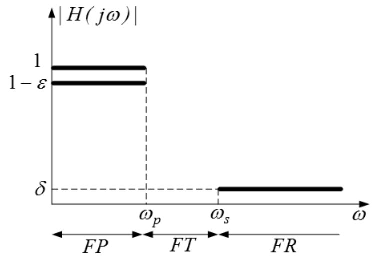
$$\omega_p = \omega_{cp}\left(\frac{\gamma}{1-\gamma}\right)^{1/2N} \qquad \omega_s = \omega_{cs}\left(\frac{1-\mu}{\mu}\right)^{1/2N}$$

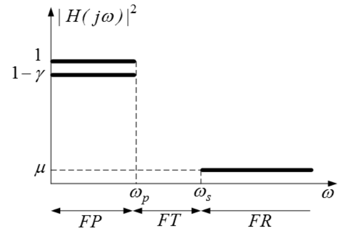
$$N = \frac{log\left(\frac{\gamma\mu}{(1-\gamma)(1-\mu)}\right)}{2log\left(\frac{\omega_p}{\omega_s}\right)}$$

TABLE I

DENOMINADOR DOS FILTROS DE BUTTERWORTH.

Ordem N	Polinônio $Q(s)$
1	$s+\omega_c$
2	$s^2+\sqrt{2}\omega_c s+\omega_c^2$
3	$s^3+2\omega_c s^2+2\omega_c^2 s+\omega_c^3$
4	$s^4+2.6131\omega_c s^3+3.4142\omega_c^2 s^2+2.6131\omega_c^3 s+\omega_c^4$
5	$s^5+3.2361\omega_c s^4+5.2361\omega_c^2 s^3+5.2361\omega_c^3 s^2+3.2361\omega_c^4 s+\omega_c^5$
6	$s^6+3.8637\omega_c s^5+7.4641\omega_c^2 s^4+9.1416\omega_c^3 s^3+7.4641\omega_c^4 s^2+3.8637\omega_c^5 s+\omega_c^6$





B. Filtros digitais

- FIR Janela retangular

$$R_d[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\Omega})e^{j\Omega n}d\Omega$$

Janela de Hamming

$$w[n]=\begin{cases}0,54-0,46\cos\left(\frac{2\pi n}{M}\right), & 0\le n\le M\\ 0, & \text{caso contrário}\end{cases}$$

- IIR

$$\omega=\frac{2}{T}\tan\left(\frac{\Omega}{2}\right)$$

Transformada bilinear

$$s=\frac{2}{T}\frac{z-1}{z+1}\qquad z=\frac{2+sT}{2-sT}$$

VI. ANÁLISE DE ESTABILIDADE

A. Critério de Routh-Hurwitz

Exemplo de construção para a equação característica:

$$Q(s)=a_4s^4+a_3s^3+a_2s^2+a_1s+a_0$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$-\frac{(a_4a_1-a_2a_3)}{a_3}=b_1$	$-\frac{(a_40-a_0a_3)}{a_3}=b_2$	$-\frac{(a_40-0a_3)}{a_3}=0$
s^1	$-\frac{(a_3b_2-a_1b_1)}{b_1}=c_1$	$-\frac{(a_30-0b_1)}{b_1}=0$	$-\frac{(a_30-0b_1)}{b_1}=0$
s^0	$-\frac{(b_10-b_2c_1)}{c_1}=d_1$	$-\frac{(b_10-0c_1)}{c_1}=0$	$-\frac{(b_10-0c_1)}{c_1}=0$

B. Critério de Nyquist $N=Z-P$

N - número de envolvimentos do ponto -1 no SH
Z - zeros de $1+G(s)H(s)$ (polos de malha fechada) no SPD
P - polos de $G(s)H(s)$ no SPD
$Z=0\rightarrow$ estável.

C. Margens de estabilidade

$$G_M|G(j\omega_M)H(j\omega_M)|=1$$
$$G_{MdB}=-20log|G(j\omega_M)H(j\omega_M)|$$
$$\Phi_M=180^o+\arg\{G(j\omega_{0dB})H(j\omega_{0dB})\}$$

ω_M - freq. onde $\angle G(j\omega)H(j\omega)=180^o$ ou -180^o.
ω_{0dB} - freq. onde $|G(j\omega)H(j\omega)|=1$.

VII. IDENTIDADES TRIGONOMÉTRICAS

$$\cos^2(x)=\frac{1}{2}[1-\cos(2x)]$$
$$\cos(a\pm b)=\cos(a)\cos(b)\mp\sin(a)\sin(b)$$
$$\sin(a\pm b)=\sin(a)\cos(b)\pm\cos(a)\sin(b)$$
$$\cos(a)+\cos(b)=2\cos(\frac{1}{2}(a+b))\cos(\frac{1}{2}(a-b))$$
$$\cos(a+b)+\cos(a-b)=2\cos(a)\cos(b)$$
$$e^{\pm j\theta}=\cos(\theta)\pm j\sin(\theta)$$
$$\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$$
$$\sin(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$
$$\text{sinc}(u)=\frac{\sin(\pi u)}{\pi u}$$

VIII. RELAÇÕES TENSÃO/CORRENTE

$$v_R(t)=Ri_R(t)$$
$$\lambda_L(t)=Li_L(t)\Rightarrow v_L(t)=L\frac{d}{dt}i_L(t)$$
$$q_C(t)=Cv_C(t)\Rightarrow v_C(t)=\frac{1}{C}\int_{-\infty}^{t}i_C(t)dt$$

IX. SÉRIE GEOMÉTRICA

Seja β um número complexo, então

$$\sum_{n=0}^{N-1}\beta^n=\begin{cases}\frac{1-\beta^N}{1-\beta}, & \beta\neq 1\\ N, & \beta=1\end{cases}$$
$$\sum_{n=0}^{\infty}\beta^n=\frac{1}{1-\beta},\qquad |\beta|<1$$