Linear Predictive Control of a Brachiation Robot

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Abstract

This work is focused on the motion control of an underactuated brachiation robot with 3 links. We present the modeling of the dynamics of the robot and introduce the application of the model-based predictive control (MPC) using a linearized model of the system. The robot has 3 revolute joints but only one of them is actuated, i. e., the robot has less control inputs than degrees of freedom. The investigation of the MPC to control such a system is due to the fact that MPC is able to deal with constraints (state or input limitations) in a direct way, obtaining an optimal control law. Also, it is investigated the use of a linearized model for prediction of system state. Simulation results are shown for complete arm switching, as well as some directions for future developments.

Keywords— *Brachiation Robot, underactuated systems, Linearized Model Predictive Control.*

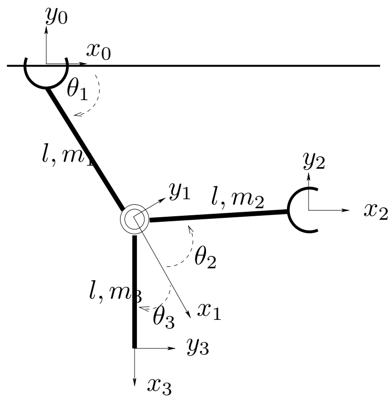

Figure 1: Three-link brachiation robot.

1 Introduction

In the beginning of the past decade a new type of robot was proposed in [1], presenting a six-link fully-actuated brachiation robot that imitates the movement of an ape swinging from branch to branch. This new type of robot effectively used the gravity for swinging, instead of just compensate for it to reduce its in¤uence. In the sequel a simpler two-link brachiation robot was proposed in [2], where a control system using CMAC (Cerebellar Model Arithmetic Computer) was proposed, considering a heuristic creation of input driving forces in a previous training. Moreover, that two-link robot was designed as an underactuated system, i. e., the robot has two degree of freedom and only one control input, so it can move only dynamically. The control of underactuated mechanical systems is a research topic that during the past years has been in vogue in the literature, for example, [3], [4], [5].

Another important work is [6], where an error learning method of the £nal-state control is applied to an underactuated robot. That paper considered a linearized state-space formulation with time-varying parameters. In [7], biomechanics arguments are considered, leading to three distinct brachiation movements, depending on the goal of the task. Those papers used a control strategy to track some reference trajectory that mimics a reference dynamical system. A fuzzy controller approach was presented in [8]. A more detailed analysis considering an underactuated brachiation robot is presented in [9].

In this paper we apply the MPC scheme to an underactuated brachiation robot, with three-links: two arms and a body. The MPC scheme is adopted because it can deal with constraints and system limits directly during the control computation [10] and generates an implicit optimal control law. Accordingly with the idea of take advantage of gravity forces to move the robot, it can be supposed that the optimal control would be one that makes effective use of such forces. Hence, the MPC would generate a control that exploit the gravity forces.

It is known that nonlinear model predictive control (NMPC) is well developed, but the main drawback of this technique is that the computational effort necessary is much higher than the linear version. The NMPC deals with on-line nonlinear optimization, which is a nonconvex problem, with a large number of decision variables and a global minimum is in general impossible to £nd [11]. The strategy proposed in this work consists in using successive linearization approach, yielding a linear, time-varying description of the system to be solved by linear MPC.

This paper is organized as follows. In section 2 is developed the nonlinear dynamic model of the brachiation robot. In section 3 the model-based predictive control scheme is presented. In the sequel (section 4) some results of simulation are presented. Discussion concerning the results and the control scheme used and some conclusions are given in section 5.

2 Underactuated Brachiation Robot Dynamics

The 3-link underactuated brachiation robot considered in this work is shown in £gure 1. This robot has two arms and one link acting like a body where payload can be located. Each arm has a gripper that can attach £rmly to the supporting line, allowing the robot to execute its movement in a way similar to a manipulator robot. It moves by releasing one arm from the supporting line and grasping it again in a forward point. It is important to keep in mind that although the robot has three joints, only one of them (joint 2) is actuated.

2.1 Nonlinear Dynamic Model

The coordinate systems attached to the robot (see £gure 1) are determined according to the Denavit-Hartenberg convention. By considering this brachiation robot as a serial open-chain robotic manipulator, its dynamics can be generally given by [12]:

$$D(\theta)\ddot{\theta}(t) + H(\theta,\dot{\theta}) + G(\theta) = \tau - F_v \quad (1)$$

Particularly for our brachiation robot, $D(\theta)$ is an 3×3 symmetric matrix representing the inertia, whose elements are:

$$\begin{aligned} d_{11} &= \frac{m_1 l^2}{3} + \frac{4m_2 l^2}{3} + m_2 l^2 \cos(\theta_2) + \\ &+ \frac{4m_3 l^2}{3} + m_3 l^2 \cos(\theta_3) \\ d_{12} &= \frac{m_2 l^2}{3} + \frac{m_2 l^2 \cos(\theta_2)}{2} \\ d_{13} &= \frac{m_3 l^2}{3} + \frac{m_3 l^2 \cos(\theta_3)}{2} \end{aligned}$$

$$\begin{aligned} d_{21} &= \frac{m_2 l^2}{3} + \frac{m_2 l^2 \cos(\theta_2)}{2} \\ d_{22} &= \frac{m_2 l^2}{3} \\ d_{23} &= 0 \end{aligned}$$

$$\begin{aligned} d_{31} &= \frac{m_3 l^2}{3} + \frac{m_3 l^2 \cos(\theta_3)}{2} \\ d_{32} &= 0 \\ d_{33} &= \frac{m_3 l^2}{3} \end{aligned}$$

$H(\theta,\dot{\theta})$ is a nonlinear Coriolis and centrifugal force vector whose elements are:

$$\begin{aligned} h_1 &= -\frac{m_2 l^2 \sin(\theta_2)\dot{\theta}_2^2}{2} - m_2 l^2 \sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 + \\ &- \frac{m_3 l^2 \sin(\theta_3)\dot{\theta}_3^2}{2} - m_3 l^2 \sin(\theta_3)\dot{\theta}_1\dot{\theta}_3 \\ h_2 &= \frac{m_2 l^2 \sin(\theta_2)\dot{\theta}_1^2}{2} \\ h_3 &= \frac{m_3 l^2 \sin(\theta_3)\dot{\theta}_1^2}{2} \end{aligned}$$

$G(\theta)$ is the gravity loading force vector whose elements are:

$$\begin{aligned} g_1 &= \frac{m_1 g l \cos(\theta_1)}{2} + \frac{m_2 g l \cos(\theta_1+\theta_2)}{2} + \\ &+ m_2 g l \cos(\theta_1) + \frac{m_3 g l \cos(\theta_1+\theta_3)}{2} + m_3 g l \cos(\theta_1) \\ g_2 &= \frac{m_2 g l \cos(\theta_1+\theta_2)}{2} \\ g_3 &= \frac{m_3 g l \cos(\theta_1+\theta_3)}{2} \end{aligned}$$

τ is the torque on joints and F_v is the diagonal matrix with viscous friction coef£cients.

Since the system is underactuated it is convenient to de£ne u such that $\tau = Pu$ with $P = [0\ 1\ 0]^T$. Then, the dynamics of the robot given by (1) can be rewritten in a more suitable state space form:

$$\dot{x} = f(x,u) \quad (2)$$

where $x = [\theta\ \dot{\theta}]^T$ is the state vector, u is the system input and

$$f(x,u) = \begin{bmatrix} \dot{\theta} \\ D(\theta)^{-1}(uP - F_v - H(\theta,\dot{\theta}) - G(\theta)) \end{bmatrix} \quad (3)$$

2.2 Linearized Dynamic Model

The linearized model around the point (x_r, u_r) is obtained by expanding the right side of (2) in Taylor series and neglecting the high order terms, resulting in:

$$\begin{aligned} \dot{x} &= f(x_r,u_r) + \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x_r\\u=u_r}} (x - x_r) \\ &+ \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x_r\\u=u_r}} (u - u_r) \end{aligned} \quad (4)$$

By subtracting $\dot{x}_r = f(x_r,u_r)$ from (4) we can write the linear model

$$\dot{\tilde{x}} = f_x\tilde{x} + f_u\tilde{u} \quad (5)$$

where f_x and f_u are the jacobians of $f(\cdot,\cdot)$ with respect to x and u, respectively, evaluated around the point (x_r,u_r), $\tilde{x} \triangleq x - x_r$ and $\tilde{u} \triangleq u - u_r$.

The approximation of $\dot{x}$ by using forward differences gives the following discrete-time system model:

$$\tilde{x}(k+1) = A(k)\tilde{x}(k) + B(k)\tilde{u}(k) \quad (6)$$

with

$$A(k) = Tf_x(kT) + I \text{ and } B(k) = Tf_u(kT) \quad (7)$$

and where T is the sampling period and k is the sampling interval.

3 MPC Control Scheme

Model predictive control is an optimal control strategy that uses the model of the system to obtain an optimal control sequence by minimizing an objective function. At each sampling interval, the model is used to predict the behavior of the system over a prediction horizon. Based on these predictions, an objective function is minimized with respect to the future sequence of inputs, thus requiring the solution of a constrained optimization problem for each sampling interval.

Although prediction and optimization are performed over a future horizon, only the values of the inputs for the current sampling interval are used and the same procedure is repeated at the next sampling time. This mechanism is known as *moving or receding horizon* strategy, in reference to the way in which the time window shifts forward from one sampling time to the next one.

3.1 MPC Problem Formulation

The basic elements present in all model-based predictive controller are: prediction model, objective function, calculation of the control action. The prediction model is the central part of the MPC, because it is important to predict the future outputs of the system. In this scheme, the state space model is used as prediction model, but in different MPC schemes, other models could be used [10]. The objective function de£nes the criteria to be optimized in order to force the generation of a control sequence that drives the system as desired.

Consider a general discrete nonlinear model, expressed as:

$$x(k+1) = f(x(k), u(k)) \tag{8}$$

where $x(k)$ is the state vector and $u(k)$ is the control input vector.

The objective function to be minimized assumes, in general, the following form:

$$\begin{aligned}\Phi(k) &= \sum_{j=N_1}^{N_2} x^T(k+j|k)\mathbf{Q}x(k+j|k) \\ &+ \sum_{j=1}^{N_u} u^T(k+j-1|k)\mathbf{R}u(k+j-1|k)\end{aligned} \tag{9}$$

where $N = N_2 - N_1$ is the prediction horizon, N_u is the control horizon and $\mathbf{Q} \geq 0$ and $\mathbf{R} \geq 0$ are weighting matrices that penalize the state error and the control effort, respectively.

By considering the fact that every real system is in practice subject to some constraint (for example physical limits), we de£ne the following general constraint expressions:

$$\begin{aligned} x(k+j|k) &\in \mathcal{X}, \quad j \in [N_1, N_2] \\ u(k+j|k) &\in \mathcal{U}, \quad j \in [0, N_u] \end{aligned} \tag{10}$$

where $\mathcal{X}$ is the closed and convex set of all possible values for x and $\mathcal{U}$ is the closed and convex set for all possible values for u. By supposing that such constraints are linear with respect to x and u, we can write:

$$\mathbf{C}x(k+j|k) \leq c, \quad j \in [N_1, N_2] \tag{11}$$

$$\mathbf{D}u(k+j|k) \leq d, \quad j \in [0, N_u] \tag{12}$$

Thus, the optimization problem, to be solved at each sample time k, is to £nd a control sequence u^* and a state sequence x^* such that minimize the objective function $\Phi(k)$ under imposed constraints, that is:

$$u^*, x^* = \arg\min_{u,x} \{\Phi(k)\} \tag{13}$$

subjected to:

$$x(k|k) = x_0 \tag{14}$$

$$x(k+j|k) = f(x(k+j-1|k), u(k+j-1|k)), \quad j \in [N_1, N_2] \tag{15}$$

$$Cx(k+j|k) \leq c, \quad j \in [N_1, N_2] \tag{16}$$

$$Du(k+j|k) \leq d, \quad j \in [0, N_u] \tag{17}$$

where x_0 is the value of x in instant k.

The problem of minimizing (13) is solved for each sampling time, resulting in the optimal control sequence:

$$u^* = \{u^*(k|k), u^*(k+1|k), \ldots, u^*(k+N_u|k)\} \tag{18}$$

and the optimal state sequence is given by:

$$x^* = \{x^*(k+N_1|k), x^*(k+N_1+1|k), \ldots, x^*(k+N_2|k)\} \tag{19}$$

with an optimal cost $\Phi^*(k)$. Thus, the control law de£ned by MPC is implicitly given by the £rst term of the optimal control sequence:

$$h(\delta) = u^*(k|k) \tag{20}$$

where $h(\delta)$ is continuous during the sampling interval T.

3.2 Brachiation Robot Control Using LMPC

The objective function takes into account the cartesian position of the end-effector of the robot, instead of considering directly the joint coordinates, because the robot must reach the supporting line ($y = 0$), independently of the joint con£guration. It is important to highlight the fact the robot is not fully actuated. Thus, the objective function is given by:

$$\begin{aligned}\Phi(t) &= \sum_{j=N_1}^{N_2} X^T(k+j|k)\mathbf{Q}X(k+j|k) \\ &+ \sum_{j=1}^{N_u} u^T(k+j-1|k)\mathbf{R}u(k+j-1|k)\end{aligned} \tag{21}$$

where $X = [x\ y]^T$ is the cartesian coordinates vector, Q is a 2×2 matrix and R is a real scalar.

The MPC deals with the system constraints as described by (11) and (12). Besides the fact that the system is underactuated, the control input has upper and lower bounds expressed by

$$-\tau_{max} \leq \tau \leq \tau_{max} \tag{22}$$

with $\tau_{max} = 30Nm$. Moreover, we have restricted the joint angle to be within the interval:

$$-\pi \leq \theta_i \leq \pi \tag{23}$$

4 Simulation Results

The objective of this work is to investigate the use of MPC to control an underactuated brachiation robot. The robot moves switching the arms as a monkey does in the branch transfer motion.

We initially posed the robot with $\theta_1 = -\pi$, $\theta_2 = 0$ and $\theta_3 = 0$ with null velocities. In this work we simulate the robot moving from the initial position to a £xed target position, such that the £nal position is in advance along X axis and the end effector achieves the supporting line $y = 0$. The prediction horizon $N = 2$ is used, with a sample interval $T = 0.01s$. The gain matrix Q is set:

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 150 \end{bmatrix} \tag{24}$$

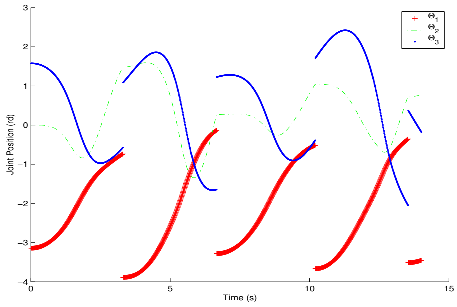

Figure 2: Angular position of each joint.

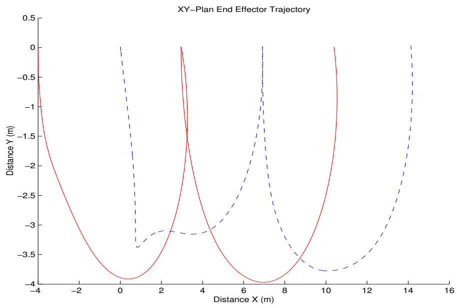

Figure 3: Trajectory on XY-plan.

and $R = 1.0$.

Figure 2 shows the angular position of each joint of the robot along the movement. The trajectory executed by the end effector in the XY-plan can be viewed in £gure 3. The unique torque applied to the system is on joint 2 and is shown in £gure 4.

5 CONCLUSIONS AND FUTURE WORKS

This work considers the application Model-based Predictive Control (MPC) to motion control of an underactuated brachiation robot with 3 links. According to the results of the previous section, it is possible to observe that the robot is able to release its hand from the supporting line and grasp it again in a forward point. It is shown that the robot is able to execute various cycles of motion, swinging and moving forward. Special attention must be taken to consider the gravitational force to reduce the external torque applied to the system, in order to maximize the time of autonomy of the robot. We intend to compare the computational effort of the linear and nonlinear model predictive control.

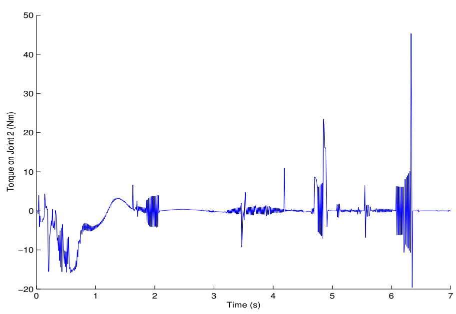

Figure 4: Torque applied on joint 2.

A real-time implementation is also planned and a more detailed analysis concerning the constraints must be carried out.

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