MPC Applied to Motion Control of an Underactuated Brachiation Robot

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Abstract— **In this paper we present the application of the Nonlinear Model based Predictive Control to the motion control of an underactuated brachiation robot. The robot has 3 links and only one joint, the second one, is actuated. The aim of this work is to develop a control input sequence that moves continuously forward the robot over an horizontal line. We investigate the use of MPC to control the underactuated brachiation robot due to its advantages, mainly the construction of an optimal stabilizing control law and due to its ability to consider in a direct way the constraints into the optimization problem. We present computational simulations with their respectives results and we highlight some important considerations.**

I. Introduction

Over the past two decades a new type of mobile robot, called Brachiation Robot, has been studied and developed. First proposed by [1], the full-actuated robot has six links and imitates the movement of an ape swinging from branch to branch. This new type of mobile robot effectively used the gravity for swinging, instead of just compensate for it. In the sequel a simpler two link brachiation robot was proposed in [2] and it was designed as an underactuated system, i. e., a system with fewer control inputs than degrees of freedom. The control of underactuated mechanical systems is a research topic that for some years has been in vogue in the literature [3]–[8]. In [9] the feedback control also considered the direction of the arm to the target. Different control strategies has been developed for controlling the brachiation robot [10], [11].

In this work we propose the application of a brachiation robot in the inspection of power transmission lines, substituting the human presence in such hazardouz task. Similar work, with different solutions, are described in [11]–[15]. The main advantage of the robot proposed in this paper is its capability to overcome obstacles on the supporting line due to its locomotion structure.

The control architecture presented in this work applies the NMPC scheme to an underactuated brachiation mobile robot, with three-links: two arms and a body. The NMPC scheme is adopted because it can deal with constraints on inputs and state and generates an implicit optimal control law [16]. If one is interested in different applications of such control scheme, see [17], [18]. Also, our approach does not suppose the existence of a reference trajectory, but it considers that the robot must achieve the horizontal line, in a position as far as possible. Furthermore, bearing in mind the idea of take advantage of gravity forces to move the robot, it can be supposed that the optimal control would be one that makes effective use of such forces. Hence, the NPMC would generate a control that exploit the gravity forces.

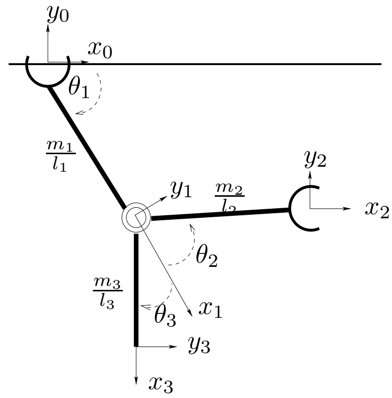

Fig. 1. Underactuated brachiation robot proposed.

This paper is organized as follows. In section II is developed the dynamic model of the brachiation robot and in section III the nonlinear model-based predictive control scheme is presented. In the sequel (sections IV e V) the control scheme proposed is explained and some results of simulation are presented. Discussion concerning the results and the control scheme used and some conclusions are given in section VI.

II. Underactuated Brachiation Robot Dynamics

The reference and coordinates systems of the robot is shown in figure 1. Considering the brachiation robot as a robotic manipulator, its dynamics can be given, in a matricial form, as:

$$D(\theta)\ddot{\theta}(t) + H(\theta,\dot{\theta}) + G(\theta) = \tau - F_v \tag{1}$$

Once we have an underactuated nonlinear system, it is suitable to define u such that $\tau \triangleq Pu$, with $P = [0\ 1\ 0]^T$.

Thus, the dynamics of the robot, defined in (1), can be described in a general state space form as:

$$\dot{q} = f(q,u) \tag{2}$$

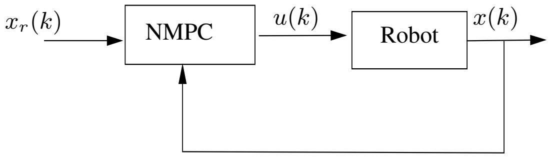

Fig. 2. NMPC scheme proposed in this work.

where $q = [\theta \ \dot{\theta}]^T$ is the state vector, u is the input of the system and

$$f(q,u) = \begin{bmatrix} \dot{\theta} \\ D^{-1}(\theta)(Pu - F_v - H(\theta,\dot{\theta}) - G(\theta)) \end{bmatrix} \quad (3)$$

The detailed development of the dynamics equations for the brachiation robot presented in this work can be seen in [19].

III. MODEL PREDICTIVE CONTROL

Model predictive control is an optimal control strategy that uses the model of the system to obtain an optimal control sequence by minimizing an objective function and according to [16] there are different implementations, but all with the same global structure. At each sampling interval, the model is used to predict the behavior of the system over a prediction horizon. Based on these predictions, an objective function is minimized with respect to the future sequence of inputs, thus requiring the solution of a constrained optimization problem for each sampling interval.

Although prediction and optimization are performed over a future horizon, only the values of the inputs for the current sampling interval are used and the same procedure is repeated at the next sampling time (*moving horizon*).

In this section we present the equations of the nonlinear model predictive control (NMPC) and then the application to the control of an underactuated brachiation mobile robot.

A. NMPC Problem Formulation

The basic elements present in all model-based predictive controller are: prediction model, objective function and optimization procedure. The prediction model is the central part of the MPC, because it is important to predict the future outputs of the system. In this scheme, the state space model is used as prediction model, but in different MPC schemes, other models could be used [20]. The objective function defines the criteria to be optimized in order to force the generation of a control sequence that drives the system as desired.

In the following we will develop the the NMPC scheme proposed in this work, represented by the block diagram in figure 2. Consider a general nonlinear model, expressed as:

$$\dot{x}(t) = f(x(t), u(t)) \quad (4)$$

where $x(t)$ is the state vector and $u(t)$ is the control input vector. The nonlinear model, now described in discrete time, is given by:

$$x(k+1) = f(x(k), u(k)) \quad (5)$$

The objective function to be minimized assumes, in general, the following form:

$$\begin{aligned} \Phi(t) &= \sum_{j=1}^{N} x^T(k+j|k)\mathbf{Q}x(k+j|k) \\ &+ \sum_{j=1}^{N} u^T(k+j-1|k)\mathbf{R}u(k+j-1|k) \end{aligned} \quad (6)$$

where N is the prediction and control horizon and $\mathbf{Q} \geq 0$ and $\mathbf{R} \geq 0$ are weighting matrices that penalize the state error and the control effort, respectively.

Considering the fact that every real system is in practice subjected to some constraint (for example physical limits), we define the following general constraint expressions:

$$\begin{aligned} x(k+j|k) &\in \mathcal{X}, \quad j \in [1, N] \\ u(k+j|k) &\in \mathcal{U}, \quad j \in [0, N] \end{aligned}$$

where $\mathcal{X}$ is the set of all possible values for x and $\mathcal{U}$ is the set for all possible values for u. By supposing that such constraints are linear with respect to x and u, we can write:

$$\mathbf{C}x(k+j|k) \leq c, \quad j \in [1, N] \quad (7)$$

$$\mathbf{D}u(k+j|k) \leq d, \quad j \in [0, N] \quad (8)$$

Thus, the optimization problem, to be solved at each sample time k, is to find a control sequence $u^\star$ and a state sequence $x^\star$ such that minimize the objective function $\Phi(k)$ under imposed constraints, that is:

$$u^*, x^* = arg \min_{u,x} \{\Phi(k)\} \quad (9)$$

subjected to:

$$x(k|k) = x_0 \quad (10)$$

$$x(k+j|k) = f(x(k+j-1|k), u(k+j-1|k)), \quad j \in [1, N] \quad (11)$$

$$Cx(k+j|k) \leq c, \quad j \in [1, N] \quad (12)$$

$$Du(k+j|k) \leq d, \quad j \in [0, N] \quad (13)$$

where x_0 is the value of x in instant k.

The problem of minimizing (9) is solved for each sampling time, resulting in the optimal control sequence:

$$u^* = \{u^*(k|k), u^*(k+1|k), \ldots, u^*(k+N|k)\} \quad (14)$$

and the optimal state sequence is given by:

$$x^* = \{x^*(k+1|k), \ldots, x^*(k+N|k)\} \quad (15)$$

with an optimal cost $\Phi^*(k)$. Thus, the control law defined by NMPC is implicitly given by the first term of the optimal control sequence:

$$h(\delta) = u^*(k|k) \quad (16)$$

where $h(\delta)$ is continuous during the sampling interval T. Hence, from the above, the closed-loop system reads:

$$\dot{x}(\delta) = f(x(\delta), h(\delta)) \quad (17)$$

IV. Brachiation Mobile Robot Control

According to the equations of the NMPC, it is necessary to describe the dynamics of the system in discrete time. Thus, the system given by (2) can be discretized using Euler method and one can rewrite:

$$q(k+1) = q(k) + Tf(k) + Tg(k)u(k) \tag{18}$$

where T is the sampling interval.

The objective function takes into account the cartesian position of the end-effector of the robot, instead of considering directly the joint coordinates, because the robot must reach the supporting line ($y = 0$), independently of the joint configuration. It is important to highlight the fact the robot is not fully actuated. Thus, the objective function is given by:

$$\begin{aligned}\Phi(t) &= \sum_{j=1}^{N} X^T(k+j|k)\mathbf{Q}X(k+j|k) \\ &+ \sum_{j=1}^{N} u^T(k+j-1|k)\mathbf{R}u(k+j-1|k) \end{aligned}\tag{19}$$

where $X = [x\ y]^T$ is the cartesian coordinates vector, Q is a 2×2 matrix and R is a real scalar.

The NMPC deals with the system constraints as described by equations (7) and (8)

$$D = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad d = \begin{bmatrix} \tau_{max} \\ \tau_{max} \end{bmatrix} \tag{20}$$

with $\tau_{max} = 30Nm$ and

$$C = \begin{bmatrix} I \\ -I \end{bmatrix}, \quad c = \begin{bmatrix} \theta_{max} \\ \theta_{max} \end{bmatrix} \tag{21}$$

where $\theta_{max} = \frac{4\pi}{3}$ is the maximum angular displacement of each joint

V. Simulation Results

In this paper we present two different computational simulations implemented using the software Matlab©. For both simulations the gains are $Q = diag(30)$, $R = 0.15$ and horizon $N = 4$. For the first simulation the angular positions are $\theta_1 = \frac{-3\pi}{4}$, $\theta_2 = -\frac{\pi}{2}$ and $\theta_3 = \frac{\pi}{4}$ and in the second simulation the robot is posed in a stretched position, with the two upper arms in the horizontal line, corresponding to the angular positions $\theta_1 = -\pi$, $\theta_2 = 0$ and $\theta_3 = \frac{\pi}{2}$. One can observe in figure 3 that around time $t = 15s$ there is a discontinuity in the angular coordinates, which occurs due to the switching of the arms. At the exact moment that the end effector reaches the horizontal line $y = 0$ closes the gripper. By changing the fixed hand of the robot, it is necessary to reassign the variable joints. Furthermore, it is important to see in figure 5 that when the arm switch happens, the angular velocities of joints 1 and 2 are zero (both hands are in contact with the horizontal line) and the last joint maintains the same angular velocity.

Once again, in figure 6 it is possible to observe a discontinuity due to the arm switching. The values of the objective

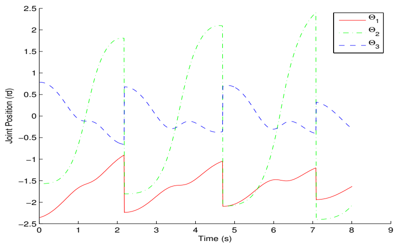

Fig. 3. Angular position of the joints on simulation 1.

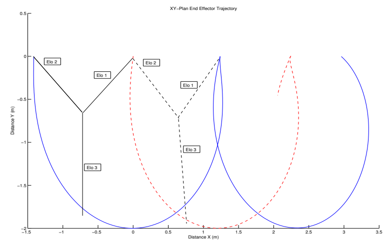

Fig. 4. Position of the end effector on plan XY.

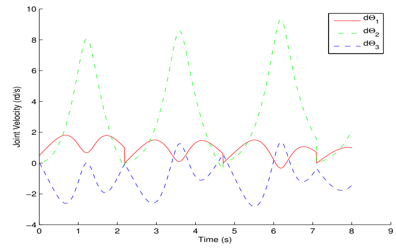

Fig. 5. Angular velocity of the joints.

function are plotted in figure 7. The arm switchings occur exactly at the minima values of the objective function.

Differently of the first simulation, the robot must swing to acquire sufficient energy to achieve the horizontal line, as presented in figure 8. Figure 9 shows the objective function. In a first look at this figure, one could ask why the switching did not occur in the minimal cost value. But after a detailed analysis it is possible to verify the minimal point occurs when the end effector crosses the horizontal line, but not in an advanced position (the axis position is still behind the fixed hand). If the end effector holds the horizontal line at this moment and releases the fixed hand, it would move backward and not forward, which is not the desired motion.

VI. Conclusion and Future Work

The fact the objective function to be minimized in the optimization process is described in terms of the cartesian position of the end effector of the robot leads one to consider the dynamics of the robot described by the cartesian coordinates of the robot. So, we intend to develop the dynamic model in cartesian coordinates to analyze the results of the simulation. Moreover, in the sequel of this work we will study the overall stability based on the Lyapunov theory (possibly using the

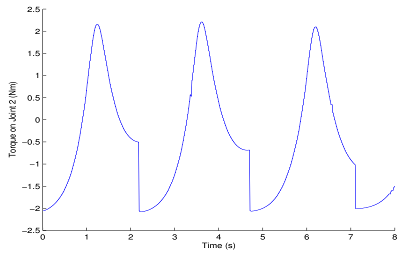

Fig. 6. Torque applied on joint 2.

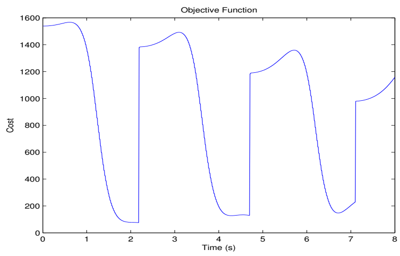

Fig. 7. Objective function on simulation 1.

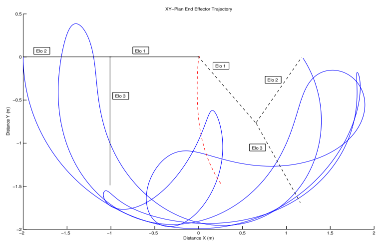

Fig. 8. Position of the end effector on plan XY.

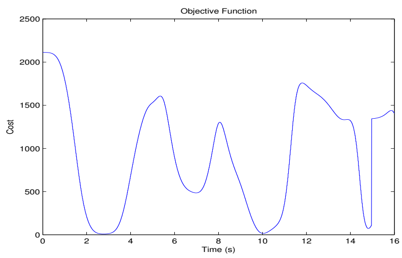

Fig. 9. Objective function on simulation 2.

contractive MPC).

In the continuation of this work we will analyze the time requirements of this control scheme, implementing the algorithms taking into acount the real time restrictions and the amount of computational effort required by the overall scheme.

VII. Acknowledgements

The authors gratefully acknowledge the financial support from CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) and from CEEE (Companhia Estadual de Energia Elétrica).

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