Ground Distance Relaying With Fault-Resistance Compensation for Unbalanced Systems

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Abstract—Fault resistance is a critical variable in distance relaying. If not considered due to underreaching phenomenon, it may cause the misoperation of ground distance relays for internal faults. Still, as a consequence of the overreaching phenomenon, the unbalanced nature of loads and asymmetry of lines can affect the distance protection operation efficiency. Mainly due to these aspects, there is low precision in protection zone limits of ground distance relays. In this paper, a new algorithm is proposed to increase the precision of these limits, improving efficiency in the distance protection process. The proposed method is based in phase coordinates and uses a fault resistance estimate to develop the trip decision procedure. The results show that the algorithm is suitable for online applications, and that it has an independent performance from the fault resistance magnitude, the fault location, and the line asymmetry.

Index Terms—Fault diagnosis, fault resistance, power system faults, power system protection, protective relaying.

I. INTRODUCTION

FAULTS are common disturbances in power systems. This phenomenon is associated with different causes, such as insulators breakdown, lightning, equipment failure, and even trees or animals in contact with electrical equipment. Due to its stochastic nature, faults are also hardly avoidable, leaving to protection engineers the task of designing protection schemes that prevent severe system damages, eliminating the fault as fast, secure, and reliable as possible. In the history of electric power systems, protection engineers have developed such designs based on different approaches, such as reliability, security, selectivity, and coordination. These designs make use of different protection equipment, such as reclosers, circuit breakers, fuses, sectionalizers, and relays [1].

Distance relaying, either phase or ground type, is frequently applied as the main protection of important transmission lines. Distance relays perform a comparison between the positive-sequence apparent impedance measured from one terminal of the line and the relay operation characteristic to decide between line tripping or not. This procedure is carried out after the fault detection, and is based on the line impedance for the trip decision [1]. During a low resistance fault, it is possible to achieve reasonable accuracy using this method, since the effective impedance between the relay and ground is close to the apparent impedance measured by the relay. Traditional distance relaying is designed to operate as primary protection (first zone, Zone 1) for a limited line impedance value. For faults outside this zone, distance relays can be used as backup protection, in time-delay coordinated stages (second and third zones) [2].

Fault resistance introduces an error in the distance estimation obtained with traditional distance relays, since in resistive faults, the distance between the relay and the fault point is not necessarily proportional to the impedance seen by the relay [1], [3]. The error introduced by the fault resistance for a symmetrical fault [1] is given by

\[
Z_A = \frac{E}{I} = Z_F + R_F \cdot \left( \frac{I_R}{I_S} + 1 \right)
\]

where \(Z_A\) is the measured apparent impedance, \(E\) and \(I\) are the voltage and current fundamental phasors calculated with data from the relay point, \(Z_F\) is the actual line impedance between the relay and the fault point, \(R_F\) is the fault resistance, and \(I_R\) and \(I_S\) are, respectively, the current phasors from the remote and the sending end of the line. The fault resistance causes an underreaching phenomenon in distance relaying, as shown in Fig. 1.

A way to overcome this problem is to compensate the fault resistance. Traditional distance relays achieve this compensation using a quadrilateral characteristic that depends on the angle between \(I_R\) and \(I_S\) [3]. By applying this technique, it is possible to obtain a better fault resistance coverage and arc compensation, without problems associated with overload misoperation of the distance relay. Other shapes of distance relay trip zones are also possible [3]–[6]. However, the fault resistance compensation is

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limited by the maximum line loading, which may cause relay misoperation for faults with high-fault resistance value.

To overcome this limitation, recent works suggest the usage of fault resistance estimation in distance relaying [7]–[11]. Those works provide a fault resistance estimate and compensation prior to the trip decision in order to accomplish better results in digital distance ground relaying. The fault resistance is estimated by using symmetrical components or modal analysis, restricting the usage of these techniques in balanced systems and equally transposed transmission lines.

In order to increase the boundaries of digital distance relaying, this paper proposes a novel fault resistance compensation technique based on phase coordinates. The estimate is achieved using one-terminal voltage and current data in an iterative process. This method is suitable for either transposed and nontransposed lines, in balanced and unbalanced systems. The method also utilizes an adaptive pickup setting and considers a constant fault resistance during the analyzed fault period. The results show the fault resistance estimate effectiveness, helping the proposed digital relay to accomplish the correct decision.

II. GROUND DISTANCE RELAYING WITH FAULT RESISTANCE COMPENSATION

In this section, a distance relaying algorithm for a line-to-ground fault is developed using phase coordinates and considering the fault resistance.

Referring to Fig. 2, which shows a single line-to-ground fault (A-g), the fault point voltage $V_{F_a}$ is given by (2)

$$V_{F_a} = V_s - x \cdot [Z_a] \cdot [I_s]$$  

(2)

where $V_s$ is the phase $a$ voltage at the relay point, $[Z_a]$ is the phase $a$ impedance vector, $x$ is the distance between the relay and the fault point, and $[I_s]$ is the relay point currents vector.

Expanding (2), (3) is obtained

$$V_{F_a} = V_s - x \cdot (Z_{aa} \cdot I_s + Z_{ab} \cdot I_s + Z_{ac} \cdot I_s)$$  

(3)

$$= R_{F_a} \cdot I_{F_a}$$  

(4)

where $R_{F_a}$ is the fault resistance and $I_{F_a}$ is the fault current, given by the relation between the sending and remote-end currents

$$I_{F_a} = I_{s_a} + I_{R_a}.$$  

(5)

Using (3)–(5), it is possible to obtain (6)

$$V_{s_a} = R_{F_a} \cdot (I_{R_a} + I_{s_a}) + x \cdot (Z_{aa} \cdot I_{s_a} + Z_{ab} \cdot I_{s_a} + Z_{ac} \cdot I_{s_a}).$$  

(6)

The apparent impedance at the sending end of the line, where the relay is located, is given by (7)

$$Z_{map} = \frac{V_{s_a}}{I_{s_a}}.$$  

(7)

Using (6) and (7), the measured apparent impedance $Z_{map}$ can be rearranged to (8)

$$Z_{map} = x \cdot \left[ Z_{aa} + \frac{Z_{ab}}{I_{s_a}} \cdot \frac{I_{s_a}}{I_{s_a}} + \frac{Z_{ac}}{I_{s_a}} \cdot \frac{I_{s_a}}{I_{s_a}} \right] + R_{F_a} \cdot \left[ 1 + \frac{I_{R_a}}{I_{s_a}} \right].$$  

(8)

From (8), the measured apparent impedance is given by two distinct components. The second term in the right side of (8) represents the fault resistance effect in the measured apparent impedance. Clearly, the fault resistance effect is not only determined by its value, but also by the relation of the sending and remote-end currents during the fault.

The first term in the right side of the equation represents the impedance of the line section between the relay point and the fault. The phase coordinates approach shows that this impedance depends on the three-phase currents, due to mutual coupling. For transposed lines and balanced loads, this term can be simplified to $x \cdot Z_{aa}$, which represents the self impedance of the faulted phase.

Typically, in nontransposed lines with a first zone setting of 85% of the line, the possible amount of overreach is 7.5% [3], which is contained in the Zone 1 setting security margin of 15%. In order to improve the robustness and precision of the proposed method, the nonfaulted phases effect must be considered.

The proposed scheme settings are:

- $\ell$ as the total line length;
- $p$ as the line length percentage to be protected;
- $[Z]$ as the line impedance matrix.

During a fault, the three-phase currents are measured online and the impedance setting is determined by the line length to be protected. With this setting, the measured impedance can be compared in order to determine the trip decision.

Rearranging (8), the final equation for the proposed ground distance relay is obtained

$$\ell \cdot p \cdot \left[ Z_{aa} + \frac{Z_{ab}}{I_{s_a}} \cdot \frac{I_{s_a}}{I_{s_a}} + \frac{Z_{ac}}{I_{s_a}} \cdot \frac{I_{s_a}}{I_{s_a}} \right] = Z_{map} - R_{F_a} \cdot \left[ 1 + \frac{I_{R_a}}{I_{s_a}} \right].$$  

(9)

During fault occurrence, the left term in (9) is determined by the measured currents and the relay settings. Then, the measured impedance $Z_{map}$ is compensated with the fault resistance estimate (whose algorithm is covered in the next section) to reach the final trip decision.

III. FAULT RESISTANCE ESTIMATION USING PHASE COORDINATES

The proposed fault resistance estimation method based in phase coordinates is an iterative process that uses online voltages and currents from the sending-end as well as the known topology of the system.

![Fig. 2. Faulted transmission line.](image-url)
A. Mathematical Development

Referring to Fig. 2, which shows an A-g fault, the voltages measured at the relay point are given by (10)

\[
\begin{bmatrix}
V_{S_a} \\
V_{S_b} \\
V_{S_c}
\end{bmatrix} = x \cdot \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \cdot \begin{bmatrix}
I_{S_a} \\
I_{S_b} \\
I_{S_c}
\end{bmatrix} + \begin{bmatrix}
V_{F_a} \\
V_{F_b} \\
V_{F_c}
\end{bmatrix}
\]

(10)

where the variables represent

- \(V_{S_a,b,c}\) sending-end phase voltages (in volts);
- \(x\) distance between the relay and the fault point (in kilometers);
- \(Z_{mn}\) phase \(m\) self impedance (in ohms per kilometer);
- \(Z_{mn}\) mutual impedance between phases \(m\) and \(n\) (in ohms per kilometer);
- \(I_{S_a,b,c}\) sending-end phase currents (in amperes);
- \(V_{F_a,b,c}\) fault point phase voltages (in volts).

For an A-g fault, the faulted phase voltage at the sending end is given by (11)

\[
V_{S_a} = V_{F_a} + x \cdot [Z_{aa}I_{S_a} + Z_{ab}I_{S_b} + Z_{ac}I_{S_c}]
\]

(11)

where \(V_{F_a} = R_F \cdot I_{F_a}\), in which \(R_F\) represents the fault resistance and \(I_{F_a}\) is the phase \(a\) fault current.

Assuming that the fault impedance is strictly resistive and constant during the analyzed period, it is possible to arrange (11) into its real and imaginary parts, resulting in (12) and (13), respectively

\[
\begin{align*}
V_{S_{ar}} &= x \cdot M_1 + R_F \cdot I_{F_ar} \\
V_{S_{ai}} &= x \cdot M_2 + R_F \cdot I_{F_ai}
\end{align*}
\]

(12) (13)

where the subscript indices \(r\) and \(i\) represent, respectively, the real and imaginary parts of the components and

\[
\begin{align*}
M_1 &= Z_{aa}I_{S_{ar}} - Z_{aa}I_{S_{ai}} + Z_{ab}I_{S_{br}} - Z_{ab}I_{S_{bi}} + Z_{ac}I_{S_{cr}} - Z_{ac}I_{S_{ci}} \\
M_2 &= Z_{aa}I_{S_{ar}} + Z_{aa}I_{S_{ai}} + Z_{ab}I_{S_{br}} + Z_{ab}I_{S_{bi}} + Z_{ac}I_{S_{cr}} + Z_{ac}I_{S_{ci}}
\end{align*}
\]

(14)

It is possible to arrange (12) and (13) into a matrix form, as

\[
\begin{bmatrix}
V_{S_{ar}} \\
V_{S_{ai}}
\end{bmatrix} = \begin{bmatrix}
M_1 & I_{F_ar} \\
M_2 & I_{F_ai}
\end{bmatrix} \cdot \begin{bmatrix}
x \\
R_F
\end{bmatrix}
\]

(16)

In (16), the sending-end voltages are a function of the fault distance and the fault resistance. It is possible to obtain (17), where the fault distance and its resistance depend on the sending-end voltages and currents as well as the line parameters \(M_1\) and \(M_2\)

\[
\begin{bmatrix}
x \\
R_F
\end{bmatrix} = \frac{1}{M_1 I_{F_{ai}} - M_2 I_{F_{ar}}} \begin{bmatrix}
I_{F_{ai}} & -I_{F_{ar}} \\
-M_2 & -M_1
\end{bmatrix} \cdot \begin{bmatrix}
V_{S_{ar}} \\
V_{S_{ai}}
\end{bmatrix}
\]

(17)

From (17), the fault resistance and the fault distance are independent and its expressions are given by (18) and (19)

\[
\begin{align*}
R_F &= \frac{I_{F_{ai}} V_{S_{ar}} - I_{F_{ar}} V_{S_{ai}}}{M_1 I_{F_{ai}} - M_2 I_{F_{ar}}} \\
\]

(18)

(19)

Based on (18) and (19), it is possible to obtain the fault distance and the fault resistance from the parameters of the system, the fault current, and the sending-end voltages. These variables are previously known, and an iterative procedure that updates the fault current is used to estimate the fault resistance.

The fault distance estimate is also used in the proposed algorithm, but not for convergence analysis. The estimate is not used since an additive error in the fault current provides higher influence in the fault distance estimate than in the fault resistance one, as proved in the Appendix.

B. Fault Current Estimation Procedure

In (19), the only unknown variable is the fault current phasor \(I_{F_{ar}}\). All other variables are the system’s parameters or measured variables.

Referring to Fig. 2, the fault current can be obtained by (20)

\[
I_{F_{a}} = I_{S_{a}} + I_{R_{a}} = I_{S_{a}} - I_{L_{a}}
\]

(20)

where \(I_{L_{a}}\) is the phase \(a\) load current.

Nevertheless, the load current during the fault period is different from the prefault load current, due to voltage drops and systems dynamics during the fault. For this reason, an iterative technique, described as follows, is used to estimate the load current during the fault [12].

Step 1) Load current \(I_{La}\) during the fault is assumed to be the same as the prefault load current.

Step 2) Fault current is calculated using (20).

Step 3) Fault location and fault resistance are estimated by (18) and (19).

Step 4) Fault point voltages are estimated using (21)

\[
\begin{bmatrix}
V_{F_a} \\
V_{F_b} \\
V_{F_c}
\end{bmatrix} = \begin{bmatrix}
V_{S_{ar}} \\
V_{S_{ai}}
\end{bmatrix} - x \cdot \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \cdot \begin{bmatrix}
I_{S_{a}} \\
I_{S_{b}} \\
I_{S_{c}}
\end{bmatrix}
\]

(21)

Step 5) Load current \(I_{L_{a}}\) is updated using the fault point voltages in (22) and (23)

\[
\begin{align*}
I_{L_{a}} &= [V_{aa} V_{ab} V_{ac}] \cdot [V_{F_a} V_{F_b} V_{F_c}]^T \\
Y_{mn} &= [(\ell - x)Z_{mn} + Z_{L_{mn}}]^{-1}
\end{align*}
\]

(22) (23)

where \(Z_{mn}\) is the line impedance (mutual or self) between phases \(m\) and \(n\) and \(Z_{L_{mn}}\) is the load impedance (mutual or self) between phases \(m\) and \(n\).

Step 6) Check whether \(R_F\) converges, using (24)

\[
|R_F(n) - R_F(n - 1)| < \delta
\]

(24)
where $\delta$ is a previously defined threshold value.

Step 7) If $R_F$ converged, stop the procedure; otherwise, go back to Step 2.

The outputs of this procedure are the fault resistance ($R_F$) and the load current ($I_{L_0} = -I_{R_0}$). These values are used in (9), along with the known values of the line impedance, the measured sending-end voltages and currents, and the proposed distance relay settings values.

IV. CASE STUDY

For the proposed method validation, several fault cases were simulated using the Alternate Transients Program (ATP)/Electromagnetic Transients Program (EMTP) [13]. The one-line diagram of the studied system is illustrated in Fig. 3. The system operates in 60 Hz and has a total line length of 8.5 km, with its impedance matrix given in (25), shown at the bottom of the page. To analyze the tests results, an admittance ($mho$) characteristic distance relay, whose first zone is set at 85% of the line impedance (a distance of 7225 m from the relay location), is used. The voltages and currents data are measured at the relay location using a sampling rate of 192 samples per cycle and the phasors are estimated using a half-cycle Fourier filter [14].

For the initial test set, the load was considered balanced and modeled as constant impedance, given by $Z_{load} = 32.4 + j10.8$ $\Omega$, for each system phase. The fault impedance trajectory for these test cases is shown in Figs. 4–8. The apparent impedance trajectories calculated with the proposed algorithm are represented by the solid line, whereas the dashed lines show the apparent impedance trajectories given by the traditional ground distance relay, using $Z_{apparent} = E_a/I_a'$, where $I_a'$ is given by (26) [1]

$$I_a' = I_a + mI_0 = I_a + \frac{Z_0 - Z_1}{Z_1} I_0$$

where $m$ is the compensation factor, $I_0$ is the zero-sequence current measured at the sending end, $Z_0$ and $Z_1$ are the zero- and positive-sequence line impedances, respectively.

Since the analyzed test case has an asymmetric line configuration, $Z_0$ and $Z_1$ are given by (27) and (28), respectively

$$Z_0 = Z_a + 2Z_{ab}$$
$$Z_1 = Z_a - Z_{ab}$$

where

$$Z_{a0} = \frac{1}{3} \cdot (Z_{aa} + Z_{ib} + Z_{cc})$$
$$Z_{ab0} = \frac{1}{3} \cdot (Z_{ab} + Z_{bc} + Z_{ca})$$

Figs. 4 and 5 show extreme fault conditions, where the fault resistance is high $R_F = 750$ $\Omega$, and the simulated fault is close to the Zone 1 limit. Fig. 4 shows an external fault (7.5 km), whereas Fig. 5 shows an internal fault (7 km) considering the first zone setting. In both cases, the proposed ground distance relay operated correctly. On the other hand, the traditional relay did not operate for the second case, in which the fault was inside Zone 1.

$$Z_{line} = \begin{bmatrix}
0.297 + j0.1858 & 0.1018 + j0.0204 & 0.0772 + j0.0038 \\
0.1018 + j0.0204 & 0.2806 + j0.1674 & 0.1018 + j0.0204 \\
0.0772 + j0.0038 & 0.1018 + j0.0204 & 0.297 + j0.1858
\end{bmatrix} \text{ } \Omega/\text{km}$$

Fig. 3. Three-phase electric power system used as a case study.

Fig. 4. Measured impedance trajectory for an A-g fault with $R_F = 750\Omega$ and $x = 7.5$ km (external fault).

Fig. 5. Measured impedance trajectory for an A-g fault with $R_F = 750\Omega$ and $x = 7$ km (internal fault).
Although the correct Zone 1 trip decision was taken in the first case by the traditional relay, a wrong Zone 2 trip decision would probably have been taken, if a second zone was set, since the measured impedance was well beyond the first zone limit. However, the proposed algorithm yields an impedance close to the first zone limit, which means that a correct Zone 2 trip decision would probably have been taken, if a second zone was set.

Figs. 6–8 present less critical situations, where faults with 4 km, 4 km, and 7.5 km from the sending end were analyzed, respectively. Once more, the proposed distance relay operated correctly for all cases. Figs. 6 and 7 show again that a traditional relay would not have operated, even for the less critical situations, where the fault is clearly inside Zone 1. Once more, the traditional distance relay would not have operated even for the hypothetical Zone 2, given the great difference between the real line impedance and the measured one. However, the proposed distance relaying scheme has operated correctly for all studied cases.

In Fig. 8, the simulated fault was outside Zone 1. However, the traditional relay operated, due to line asymmetry, which affects the relay by overreaching its protection zones. In this situation, the proposed scheme did not operate since the trip decision is taken only after the algorithm converges to a fault resistance value. Hence, even if the algorithm yields an apparent impedance inside the protected zone during the iterative procedure, it will not operate until the algorithm converges, which is a decision defined by (24). This case shows that the algorithm defines the protection zones limits very precisely.

The second set of simulations were cases with highly unbalanced loads, given by $Z_{kwha} = 32.4 + j10.8 \ \Omega$, $Z_{kwhb} = 25.92 + j8.64 \ \Omega$, and $Z_{kwhc} = 20.16 + j9.72 \ \Omega$. The results are shown in Figs. 9–13.

Fig. 9 shows a less critical case, similar to the test presented in Fig. 8, but with an unbalanced load. Once more, the traditional relay operated incorrectly, whereas the proposed distance relay did not operate, which was the correct decision. In this case, the algorithm also yields an apparent impedance inside the protected zone; however, the trip decision is only made when the algorithm converges for a fault resistance value, yielding an apparent impedance outside the protected zone.

Figs. 10 and 11 represent critical situations with unbalanced loads, where faults were simulated at a distance of $R_F = 500 \ \Omega$, 7 km, and 7.5 km from the relay point, respectively. For both cases, the proposed relay operated correctly, whereas the traditional relay misoperated for the internal fault in Fig. 10, due to underreaching phenomenon caused by the fault resistance.
Again, the traditional relay would not have operated even in Zone 2 for the second case, due to the high error in the measured impedance component compared to the actual line fault impedance.

Figs. 12 and 13 show the impedance trajectory for faults with a distance of $R_F = 40 \, \Omega$, 3.5 km, and 7.5 km from the sending end, respectively. In both situations, the compensated relay operated correctly again. Once more, a traditional relay would not have operated for Zone 1. However, for Zone 2 or Zone 3, it might have operated, according to the relay setting. The different behavior for this case is due to the relatively small fault resistance.

Table I shows an overview of the simulations results. Columns one and two show, respectively, the case number and the fault distance to the sending end. Columns three and four show the apparent impedance measured by the proposed algorithm and the traditional ground relay, respectively. The Zone 1 decision for the proposed and the traditional relays are shown in columns five and six. The fault resistance simulated and estimated value as well as its relative error are shown in columns seven to nine. Columns ten and eleven show, respectively, the number of iterations required by the fault resistance estimation algorithm to converge, given by $N$ and the computational time taken by the algorithm (measured in Matlab using a PC Celeron 2.4 GHz and 480-Mb RAM).

V. RESULTS ANALYSIS

In this section, the results are analyzed in different aspects: fault resistance, fault distance, load, and system’s unbalance effect, as well as the computing time required by the proposed methodology.

A. Fault Resistance

Referring to Table I and comparing the fault resistance estimate error, for the case groups 1–5 and 3–4 (each pair has the same systems conditions and fault distances, while the fault resistance varies), it is possible to verify that the fault resistance estimation errors were not significantly affected by the resistance value itself. As the fault resistance increases, the error also increases. However, the effect is not significant, since the highest error was 0.19% for the extreme case of $R_F = 750 \, \Omega$. Similar results were obtained with unbalanced load tests, considering cases 6, 8, and 10.
Also, comparing the pairs of cases 1–5, 3–4, and 8–10, it is verified that as the fault resistance increases, the number of iterations required for convergence also increases. The convergence process requires 35 iterations for the extreme unbalanced load case of a fault with a distance of $R_F = 500 \, \Omega$ and 7.5 km from the relay location. The same conclusions apply to the computing time required to reach the trip decision, since it is proportional to the iterations number.

B. Fault Distance

The fault distance effect can be analyzed by the results of the case groups 1-2-3, 7–8, and 9–10 (each group has the same system conditions and fault resistance, while the fault location varies). In all of these groups, except 9–10, it is possible to verify that there is a small effect of the fault distance in the fault resistance estimation procedure. It is a very low influence compared to the fault resistance one. The highest variation verified was 0.02% for the pair 1–3. Also, as the fault resistance increases, the fault distance effect also increases. This can be verified by analyzing the fault resistance mean error estimate for each group 1-2-3, 7–8, and 9–10.

Analyzing the same case groups 1-2-3, 7–8, and 9–10, it is verified that as the fault resistance increases, the fault distance starts to affect the computational process. The difference in the number of iterations is very low for the same fault resistance and different fault distance, a fact that leads to the conclusion that the fault distance has a very low effect in the computational process of the proposed numerical algorithm.

C. Load and System’s Unbalance Effect

It is clear from Table I and the other analyzed aspects, that the load and system’s unbalance only affect the computing time required by the algorithm. The fault resistance estimate is clearly not affected by this aspect, as the overall performance of the proposed distance relay shows. With an unbalanced load, the computing time process increases, as it is shown by comparing the pair cases 1–8 and 2–7. Even for the fault situations with a balanced load and $R_F = 750 \, \Omega$ (cases 1 and 2), the required computing time for the unbalanced load fault cases (7 and 8) with $R_F = 500 \, \Omega$ is higher, which shows the effect of the load unbalance only to the required computational time.

D. Computational Time

As can be seen in Table I, the required processing time is reasonable for practical applications. The time required in the studied cases never reached beyond 11 ms, which represents less than 1 cycle for the 60-Hz frequency. The total processing time required to reach a final trip decision is the time shown in Table I, added by the data-acquisition time. The data acquisition time is Fourier algorithm dependent. In this work, a half-cycle Fourier algorithm [14] is used. The maximum total processing time obtained was 20 ms. Faster applications could use a quarter-cycle Fourier algorithm.

The proposed methodology was implemented in a conventional desktop PC, which performs several tasks concomitantly, which increases the total processing time. In practical situations, dedicated hardware would perform this task, enhancing the performance of the digital algorithm in this aspect. It is important to note that the iterations number probably would not be affected by a dedicated hardware, only by the total processing time.

VI. Conclusion

In this paper, a new numerical distance relaying algorithm based on phase coordinates and fault resistance estimation is proposed. The method is suitable for balanced and unbalanced systems, including systems with transposed lines and balanced loads as well as systems with high line asymmetry and unbalanced loads.

The results show that the proposed methodology is not significantly affected by the fault resistance or the fault location. Therefore, it operates correctly even in extreme situations: faults with high resistance close to the zone limits, with unbalanced loads and line asymmetry. The results also show that the methodology is robust in these aspects with a precise zone limit definition. Also, the computational time required is suitable for practical implementation even in fast protection systems designs.

APPENDIX

ERROR INFLUENCE IN THE FAULT DISTANCE AND RESISTANCE EQUATIONS

To visualize the influence of an additive error in the fault current estimate, it is possible to consider the fault current value as

\[
I_{F_{\text{mea}}} = I_{F_{\text{est}}} + \epsilon_1 \tag{31}
\]

\[
I_{F_{\text{est}}} = I_{F_{\text{ref}}} + \epsilon_2 \tag{32}
\]

in (18) and (19), where $\epsilon_1$ and $\epsilon_2$ are the fault current real and imaginary part errors, respectively.
The results are given by (33) and (34)

\[
x = \frac{I_{F_{a1}} V_{S_{a1}} - I_{F_{a2}} V_{S_{a2}} + \left(\frac{\varepsilon_2 V_{S_{a2}} - \varepsilon_1 V_{S_{a1}}}{M_1 I_{F_{a1}} - M_2 I_{F_{a2}}} + \frac{\varepsilon_2 M_1 - \varepsilon_1 M_2}{M_1 I_{F_{a1}} - M_2 I_{F_{a2}}}\right)}{M_1 I_{F_{a1}} - M_2 I_{F_{a2}}}
\]

(33)

\[
R_F = \frac{-M_2 V_{S_{a2}} + M_1 V_{S_{a1}}}{M_1 I_{F_{a1}} - M_2 I_{F_{a2}} + \left(\frac{\varepsilon_2 M_1 - \varepsilon_1 M_2}{M_1 I_{F_{a1}} - M_2 I_{F_{a2}}}\right)}
\]

(34)

which can be expressed, respectively, by (35) and (36)

\[
x = \frac{I_{F_{a1}} V_{S_{a1}} - I_{F_{a2}} V_{S_{a2}} + \frac{\varepsilon_1}{\beta} V_{S_{a1}}}{\beta}
\]

(35)

\[
R_F = \frac{-M_2 V_{S_{a2}} + M_1 V_{S_{a1}}}{\beta}
\]

(36)

where

\[
\alpha = \varepsilon_2 V_{S_{a2}} - \varepsilon_1 V_{S_{a1}}
\]

(37)

\[
\beta = M_1 I_{F_{a1}} - M_2 I_{F_{a2}} + \varepsilon_2 M_1 - \varepsilon_1 M_2
\]

(38)

From (35)–(38), it is possible to conclude that the error in the fault current estimate causes an equal percentage error in the calculation of both variables \(x\) and \(R_F\), given by \(1/\beta\). However, in the fault distance equation, there is one more term provided by the fault current estimate error, given by \(\alpha/\beta\).

Therefore, an additive error affects the effective fault distance estimate more significantly compared to the fault resistance one. In this way, the use of the fault resistance in the algorithm, and not directly the fault location, for the distance estimation, can provide better performance for the algorithm. This justifies the application of the fault resistance in the proposed method instead of the fault location directly.

REFERENCES


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